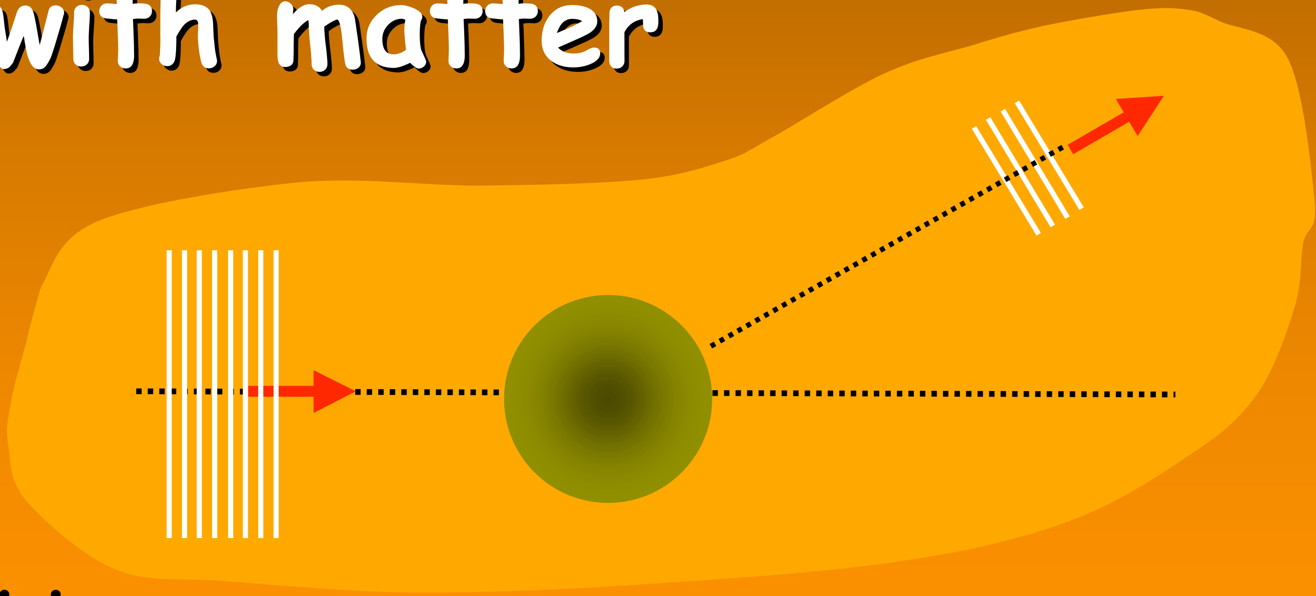
Interaction of X-rays with matter



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Overview

- Attenuation of x-rays
  - Classical Thomson scattering
  - Basic interference phenomenon
  - Scattering: deviations from classical behaviour
  - Photoelectric absorption

Attenuation of X-rays

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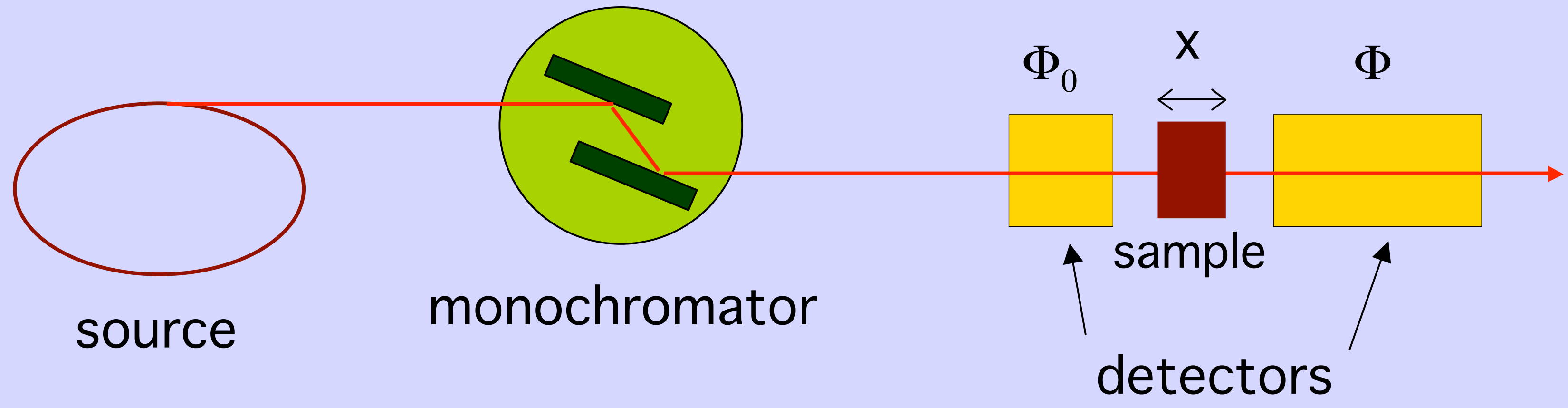

Exponential attenuation

$$\Phi = \Phi_0 \exp\left[-\mu(\omega)\, x\right]$$

Attenuation coefficient

$$\mu(\omega) = \frac{1}{x} \ln \frac{\Phi_0}{\Phi}$$

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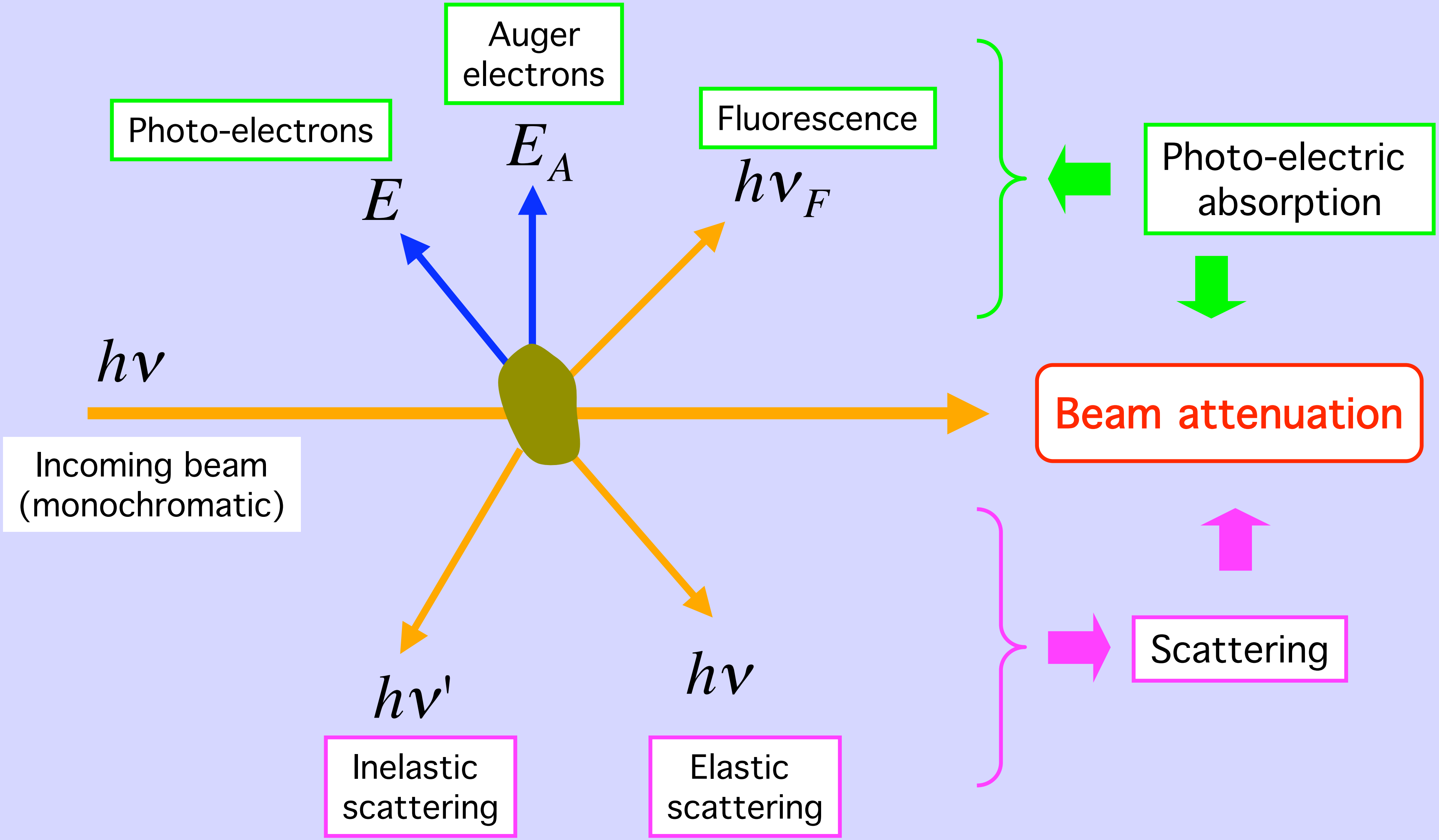

Structural techniques

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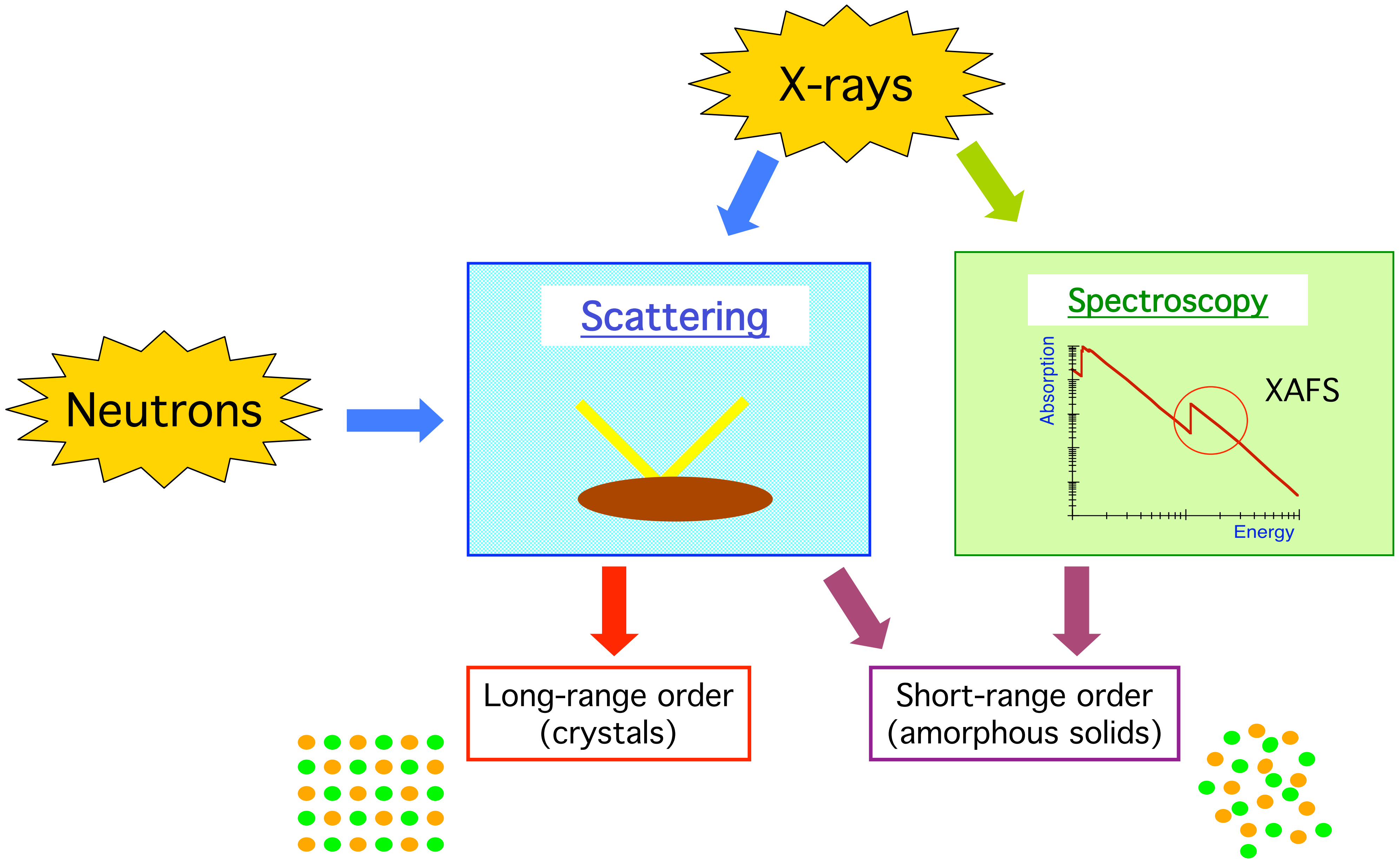

- Scattering from a free charge
  classical treatment (Thomson scattering)

Charge acceleration

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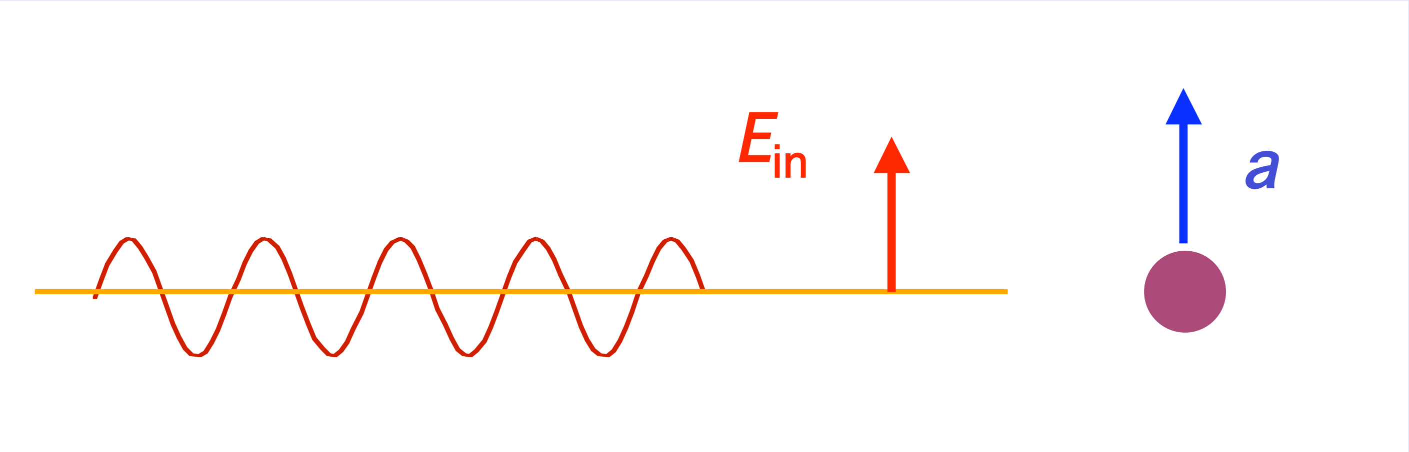
Incoming electric field

$$\vec{E}_{\text{in}}(t) = \vec{E}_0 \cos(\omega t)$$

Charge acceleration

$$\vec{a}(t) = \frac{q}{m} \vec{E}_0 \cos(\omega t)$$



Magnetic field effects are negligible

Electrons .vs. protons

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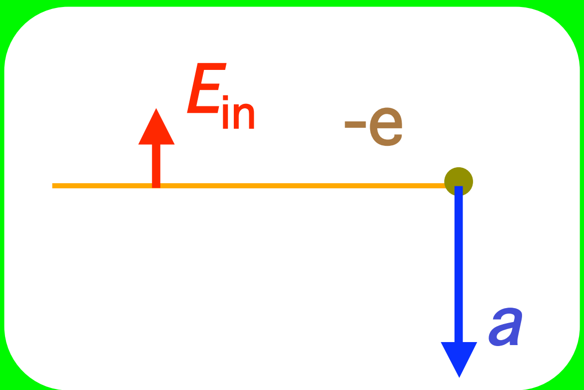
$$\vec{a}(t) = \frac{q}{m}\vec{E}_0 \cos(\omega t)$$

Electron $q = -e$

$$\frac{e}{m} \approx 1.7 \times 10^{11} \text{ C/kg}$$

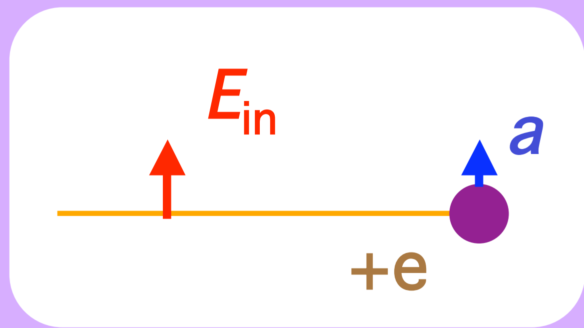


π phase-shift

$$\frac{M}{m} \approx 1836$$

Proton $q = +e$

$$\frac{e}{M} \approx 0.96 \times 10^{8} \text{ C/kg}$$



Dipole emission of radiation

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Accelerating charge ⇒ electromagnetic field

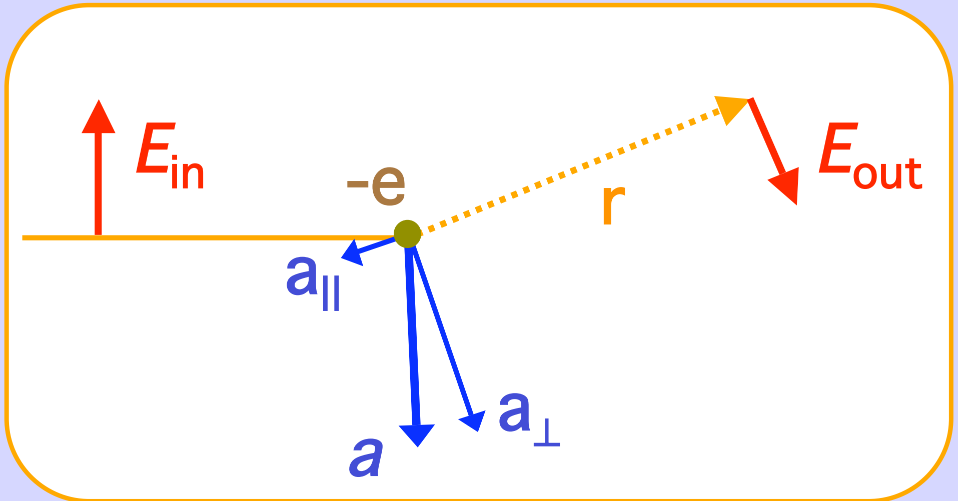
Dipole approximation:

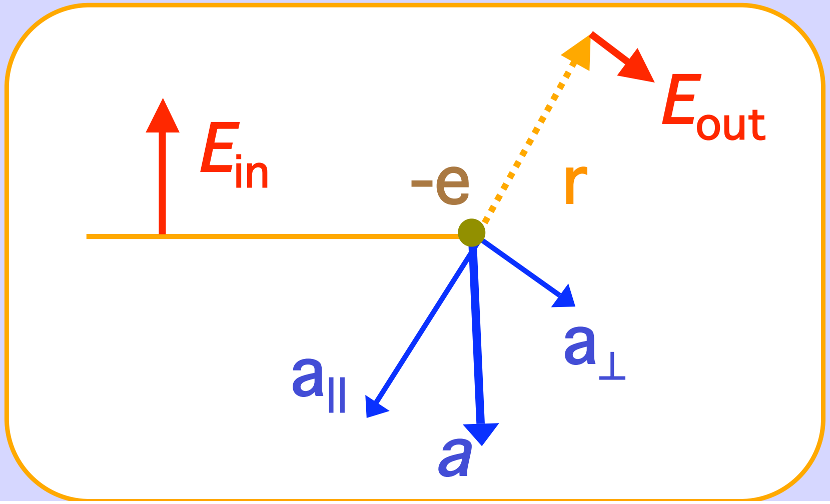
- charge velocity: v << c
- charge distribution: d << λ
- observer distance: r >> λ

$$\vec{a}(t) = -\frac{e}{m}\vec{E}_0 \cos(\omega t)$$

Electron: $q = -e$

$$\vec{E}_{out}(\vec{r},t) = \frac{e\,\vec{a}_\perp(t')}{4\pi\varepsilon_0\, r\, c^2}$$





Outgoing electric field

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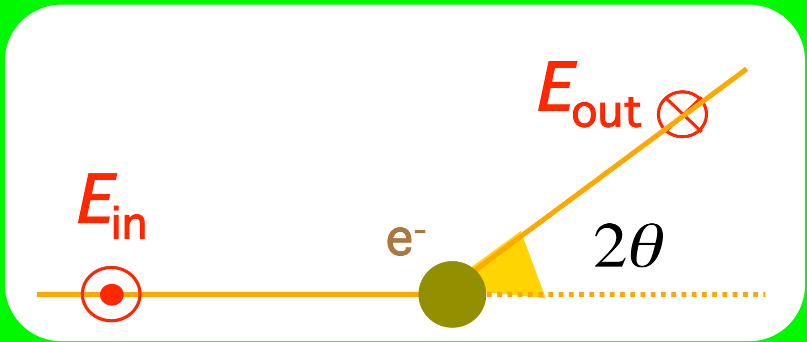
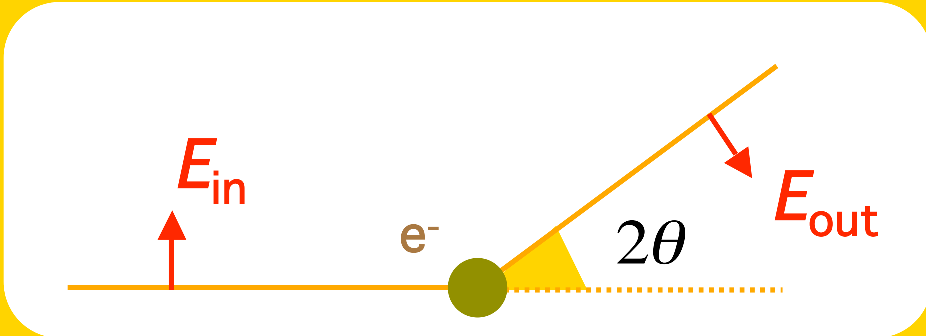
π polarisation



$$E_{out}(\vec{r},t) = -\frac{r_e}{r} E_0 \cos(\omega t')$$

σ polarisation



$$E_{out}(\vec{r},t) = -\frac{r_e}{r} E_0 \cos(\omega t') \cos(2\theta)$$

$$r_e = \frac{e^2}{4\pi\varepsilon_0 c^2 m}$$

Thomson scattering length

or

Classical electron radius

Thomson scattering length

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= Classical electron radius

Energy of a uniformly charged:

- spherical shell $$U = \frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r_e}\right)$$
- spherical volume $$U = \frac{3}{5}\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r_e}\right)$$

Electron

$$U \approx \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_e} = mc^2$$

Relativistic rest energy

$$r_e = \frac{e^2}{4\pi\varepsilon_0 c^2 m} = 2.8\text{x}10^{-15}\,\text{m} = 2.8\text{x}10^{-5}\,\text{Å}$$

Radiated power

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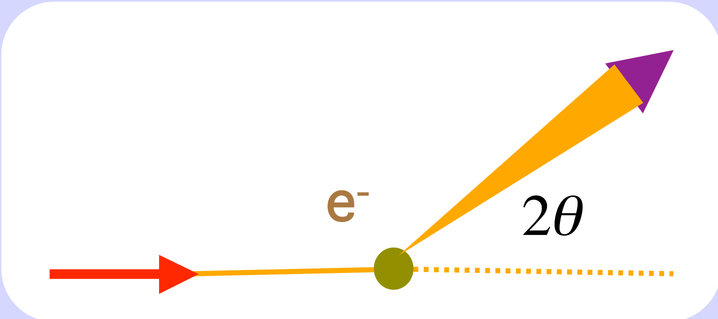
Incoming power flux

$$I_0 = \frac{1}{2}\varepsilon_0 E_0^2 c \quad [\mathrm{W/m^2}]$$

Un-polarized beam



Emitted power

$$P(\theta)\,d\theta = r_e^2 \left[\frac{1+\cos^2(2\theta)}{2}\right] I_0\; d\theta$$

Polarisation factor

Independent from radiation wavelength

Polarisation factor

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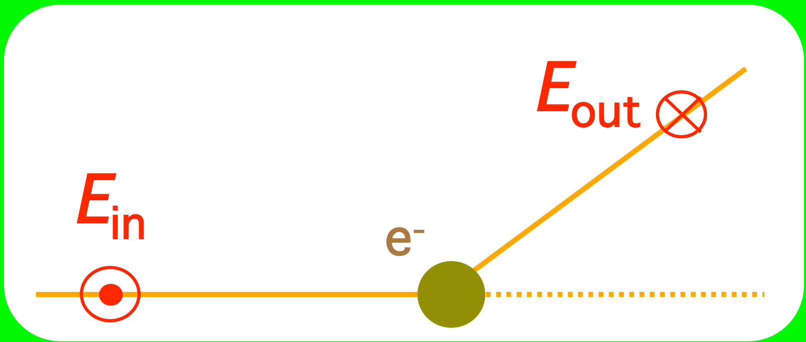
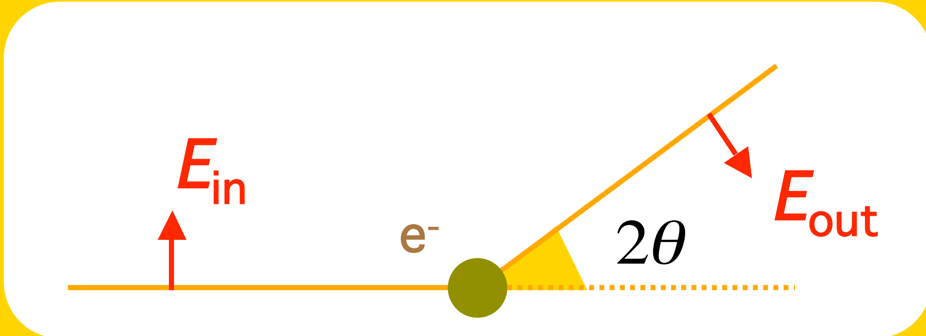
π polarisation



σ polarisation



Un-polarised beam

$$\left[\frac{1}{2}+\frac{\cos^2(2\theta)}{2}\right]$$

Laboratory x-ray sources

Total electron cross-section

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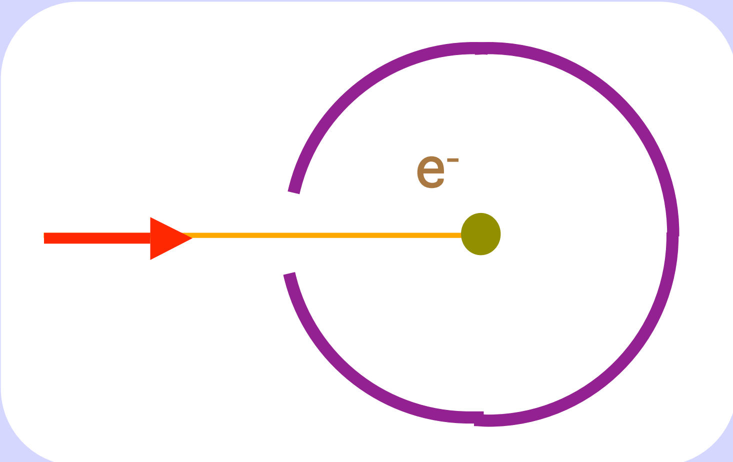
Incoming power flux

$$I_0 = \frac{1}{2}\varepsilon_0 E_0^2 c \quad \left[\mathrm{W/m^2}\right]$$

Un-polarized beam



Total radiated power

$$P = \frac{8}{3}\pi r_e^2 \, I_0$$

Electron cross-section

$$\sigma_e = 6.66\mathrm{x}10^{-29}\,\mathrm{m}^2$$

Independent from radiation wavelength

Beyond classical treatment

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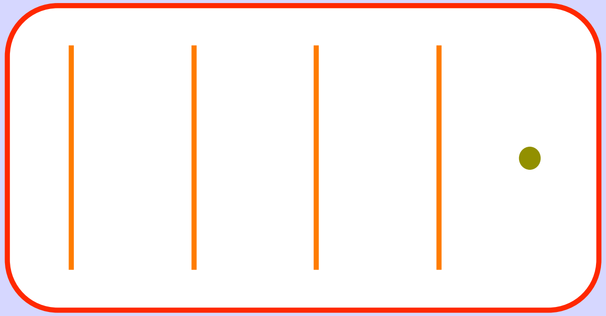
Thomson scattering:



Free electron

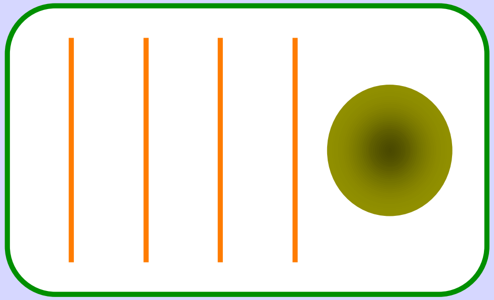
Elastic scattering ☹

$r_e << \lambda$

Free electron ⇒ Inelastic scattering (Compton)

Electrons are bound in atoms

Probabilistic distribution of e^- charge



Interference

Interference

Scattering from many electrons

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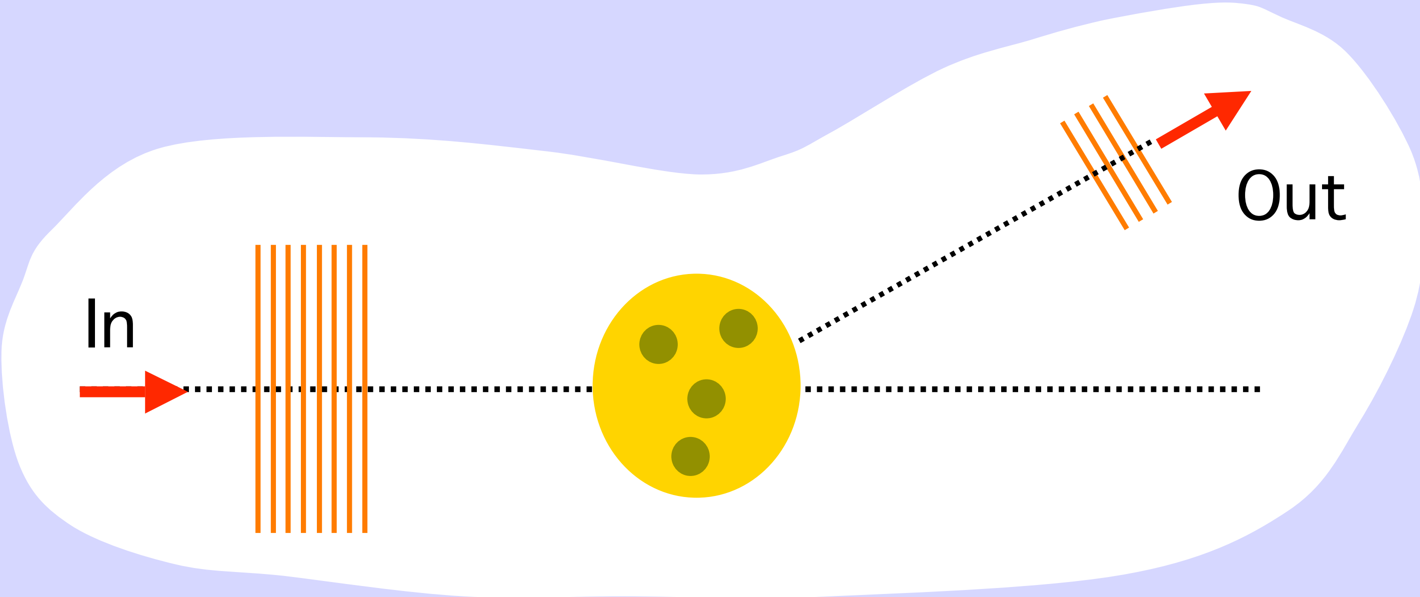

Waves from different electrons → **Interference**

Depending on radiation wavelength

Scattering vector

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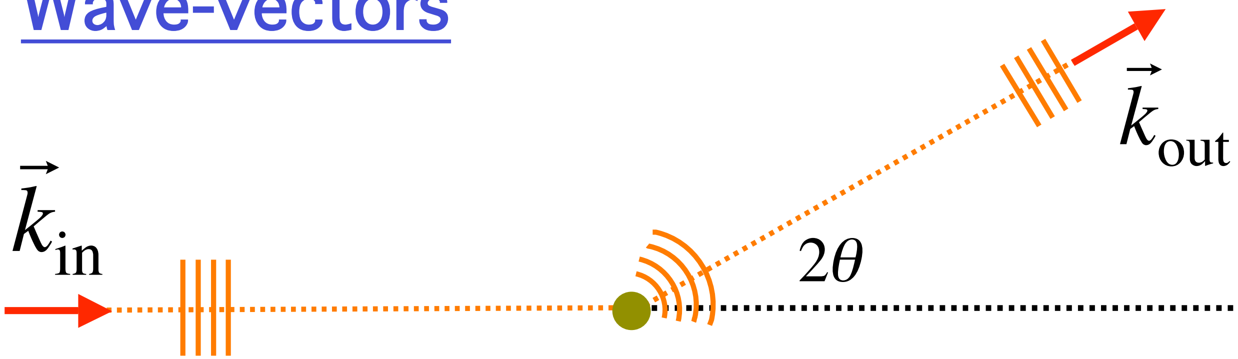
Wave-vectors



$$\vec{k}_{\text{in}} = \frac{2\pi}{\lambda}\hat{s}_{\text{in}}$$

$$\vec{k}_{\text{out}} = \frac{2\pi}{\lambda}\hat{s}_{\text{out}}$$

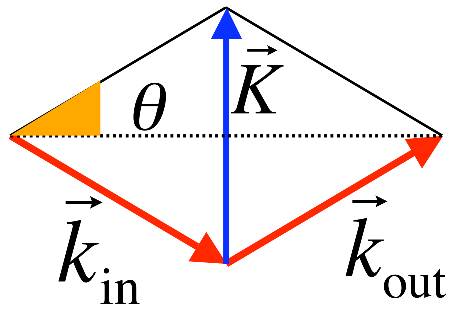
Elasting scattering

$$\left|\vec{k}_{\text{in}}\right| = \left|\vec{k}_{\text{out}}\right| = \frac{2\pi}{\lambda}$$

Scattering vector

$$\vec{K} = \vec{k}_{\text{out}} - \vec{k}_{\text{in}}$$

$$\left|\vec{K}\right| = 4\pi\frac{\sin\theta}{\lambda}$$



Incoming wave

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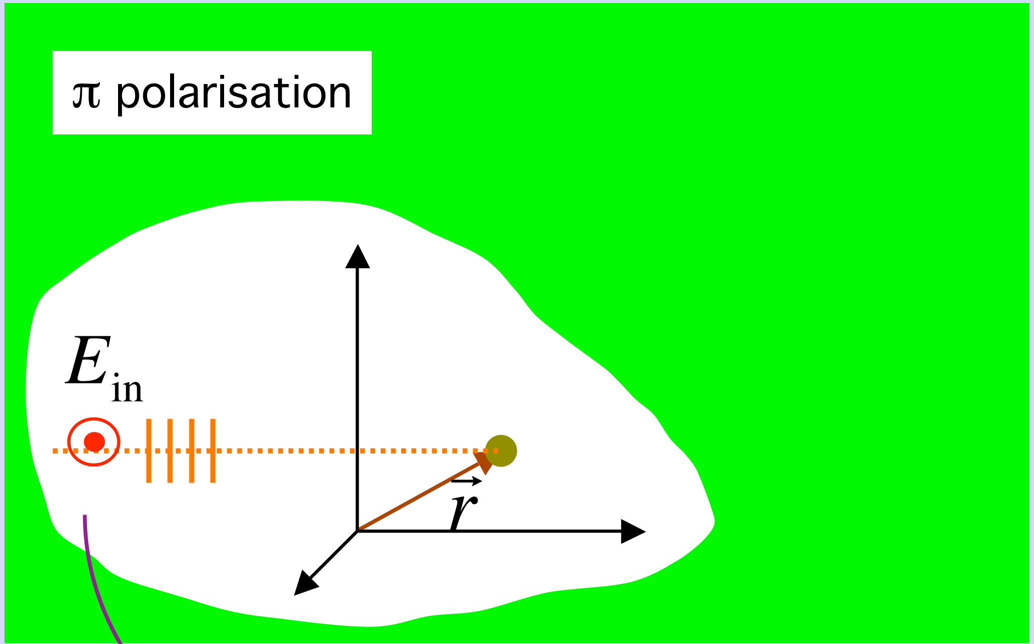

$$E_{in} = E_0 \cos\left(\omega t - \vec{k}_{in} \cdot \vec{r}\right)$$

$$= \mathrm{Re}\left\{ E_0 e^{i\left(\omega t - \vec{k}_{in} \cdot \vec{r}\right)} \right\}$$

Complex notation

A_0 Input amplitude

Outgoing wave

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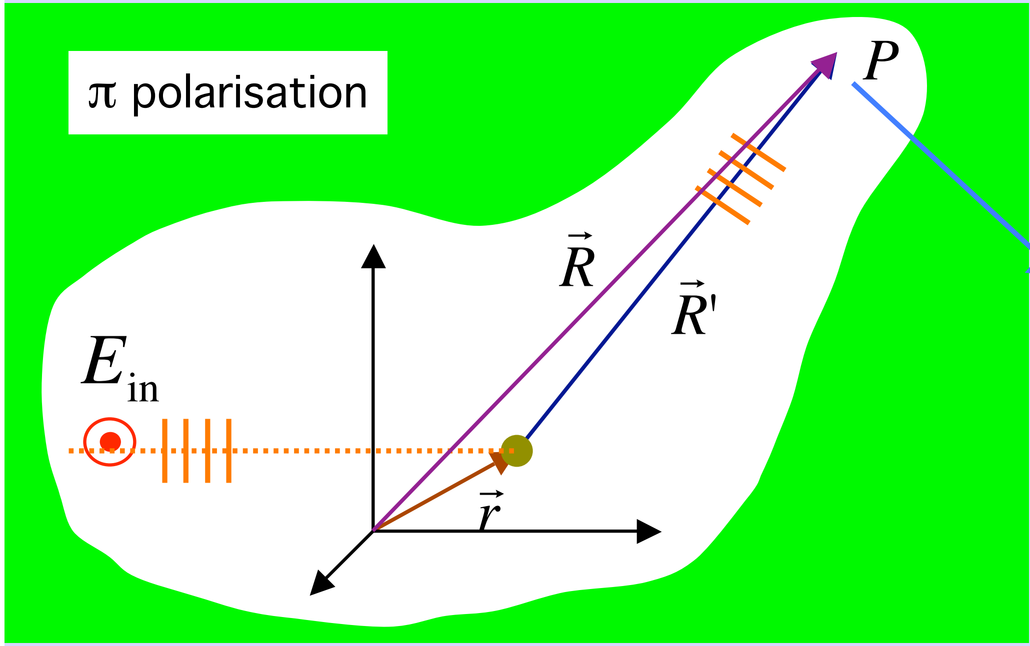

$$\overbrace{\phantom{E_0 e^{i(\omega t - \vec{k}_{in}\cdot\vec{r})}(-r_e)}}^{A_0}$$

$$E_{out} = \mathrm{Re}\left\{ \underbrace{E_0 e^{i(\omega t - \vec{k}_{in}\cdot\vec{r})} (-r_e) \frac{e^{-ik_{out}R'}}{R'}}_{A} \right\}$$

A_0 = $E_0 e^{i(\omega t - \vec{k}_{in}\cdot\vec{r})} (-r_e)$

A

Scattered amplitude

Geometrical tricks

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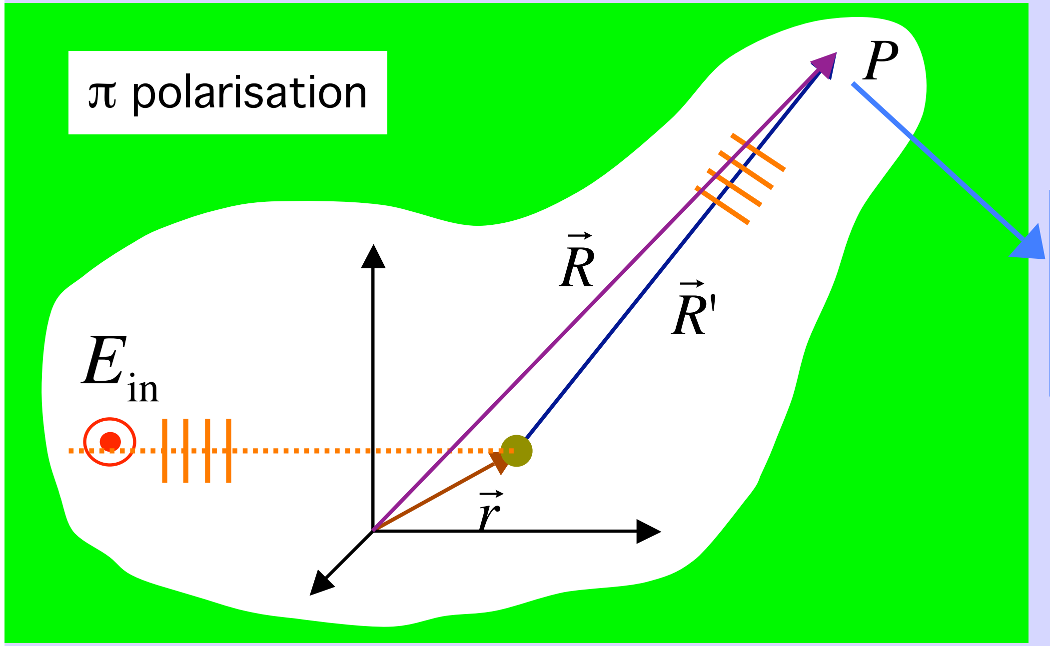

$$E_{\text{out}} = \text{Re}\left\{E_0 e^{i(\omega t - \vec{k}_{\text{in}}\cdot\vec{r})}\,(-r_e)\,\frac{e^{-ik_{\text{out}}R'}}{R'}\right\}$$

$$\frac{e^{-ik_{\text{out}}R'}}{R'} \approx \frac{e^{-ik_{\text{out}}R}}{R}\, e^{ik_{\text{out}} r\cos(\vec{k}_{\text{out}}\cdot\vec{r})} = \frac{e^{-ik_{\text{out}}R}}{R}\, e^{i\vec{k}_{\text{out}}\cdot\vec{r}}$$

Scattered amplitude

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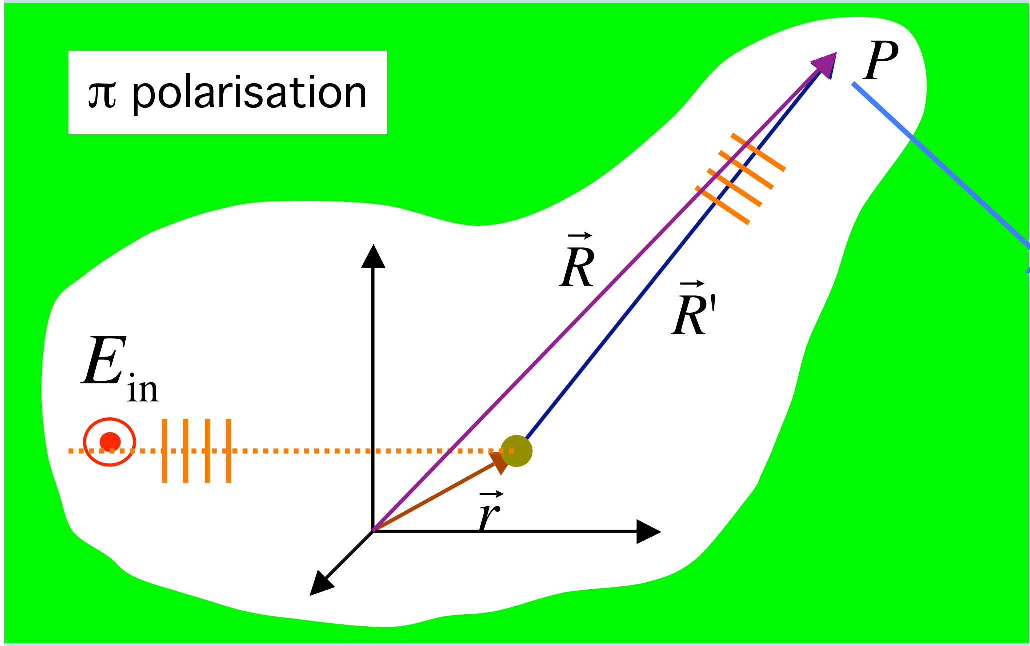

$$E_{out} = \mathrm{Re}\left\{E_0 e^{i(\omega t - \vec{k}_{in}\cdot\vec{r})} (-r_e) \frac{e^{-ik_{out}R'}}{R'}\right\}$$

Scattered amplitude

$$A(\vec{K}) = -E_0\, r_e \frac{e^{-ik_{out}R}}{R}\, e^{i\omega t}\, e^{i(\vec{k}_{out}-\vec{k}_{in})\cdot\vec{r}} \rightarrow e^{i\vec{K}\cdot\vec{r}}$$

Phase factor

Amplitude and intensity

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$$A(\vec{K}) = -E_0\, r_e\, \frac{e^{-ik_{\text{out}}R}}{R}\, e^{i\omega t}\, e^{i\vec{K}\cdot\vec{r}}$$

(π polarisation)

$$E_{\text{out}} = \mathrm{Re}\left\{A\left(\vec{K}\right)\right\}$$

Cannot be measured !

We measure:

$$I = \left|A(\vec{K})\right|^2 = \begin{cases} E_0^2\, r_e^2\, \dfrac{1}{R^2} & \text{(π polarisation)} \\[2ex] E_0^2\, r_e^2\, \dfrac{1}{R^2}\left[\dfrac{1+\cos^2(2\theta)}{2}\right] & \text{un-polarized beam} \end{cases}$$

Basic interference effect (a)

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1 electron (π polarisation)

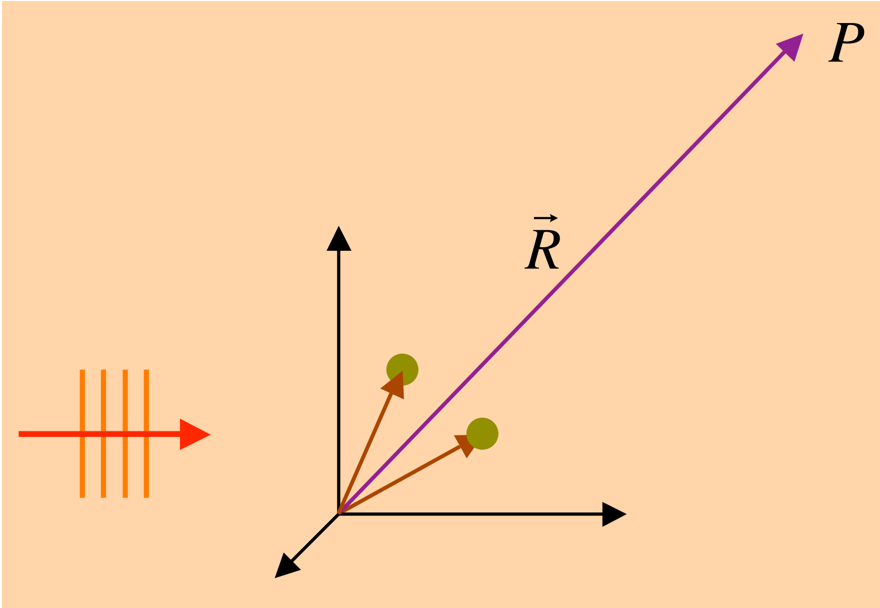
$$A(\vec{K}) = -E_0 \, r_e \, \frac{e^{-ik_{\text{out}}R}}{R} \, e^{i\omega t} \, e^{i\vec{K}\cdot\vec{r}}$$

$$= A_{\text{el}} \, e^{i\vec{K}\cdot\vec{r}}$$

2 electrons (π polarisation)

$$A(\vec{K}) = A_1(\vec{K}) + A_2(\vec{K})$$

$$= A_{\text{el}} \left[e^{i\vec{K}\cdot\vec{r}_1} + e^{i\vec{K}\cdot\vec{r}_2} \right]$$



Basic interference effect (b)

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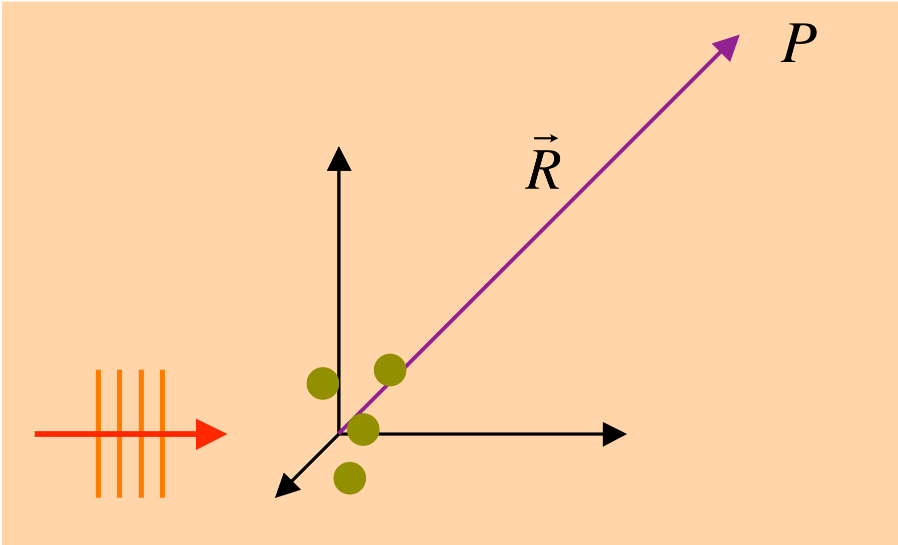
n electrons (π polarisation)

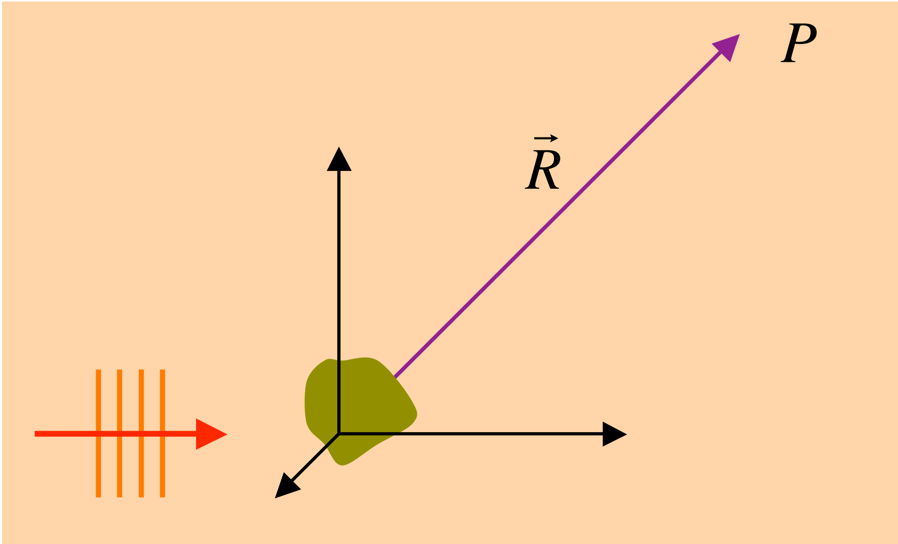
$$A(\vec{K}) = A_{\text{el}} \sum_i e^{i\vec{K}\cdot\vec{r}_i}$$



Continuous charge distribution

(ρ = number density)

$$A(\vec{K}) = A_{\text{el}} \int \rho_e(\vec{r})\, e^{i\vec{K}\cdot\vec{r}}\, dV$$



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The scattered amplitude is the Fourier transform of the electron density

$$A(\vec{K}) = A_{\mathrm{el}} \int \rho_e(\vec{r})\, e^{i\vec{K}\cdot\vec{r}}\, dV$$

(number density)

Amplitude and intensity (b)

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Amplitude

$$A(\vec{K}) = A_{\text{el}} \int \rho_e(\vec{r})\, e^{i\vec{K}\cdot\vec{r}} dV$$

Cannot be measured !

Intensity

$$I(\vec{K}) = \left|A(\vec{K})\right|^2 = \left|A_{\text{el}}\right|^2 \left|\int \rho_e(\vec{r})\, e^{i\vec{K}\cdot\vec{r}} dV\right|^2$$

Is measured,
but
phase information
is lost !

Structural info

$$\left|A_{\text{el}}\right|^2 = r_e^2\, E_0^2\, \frac{1}{R^2} \left[\frac{1+\cos^2(2\theta)}{2}\right]$$

Polarisation factor
for unpolarised beam

Deviations from classical treatment

a) Electronic distribution

Electronic orbitals

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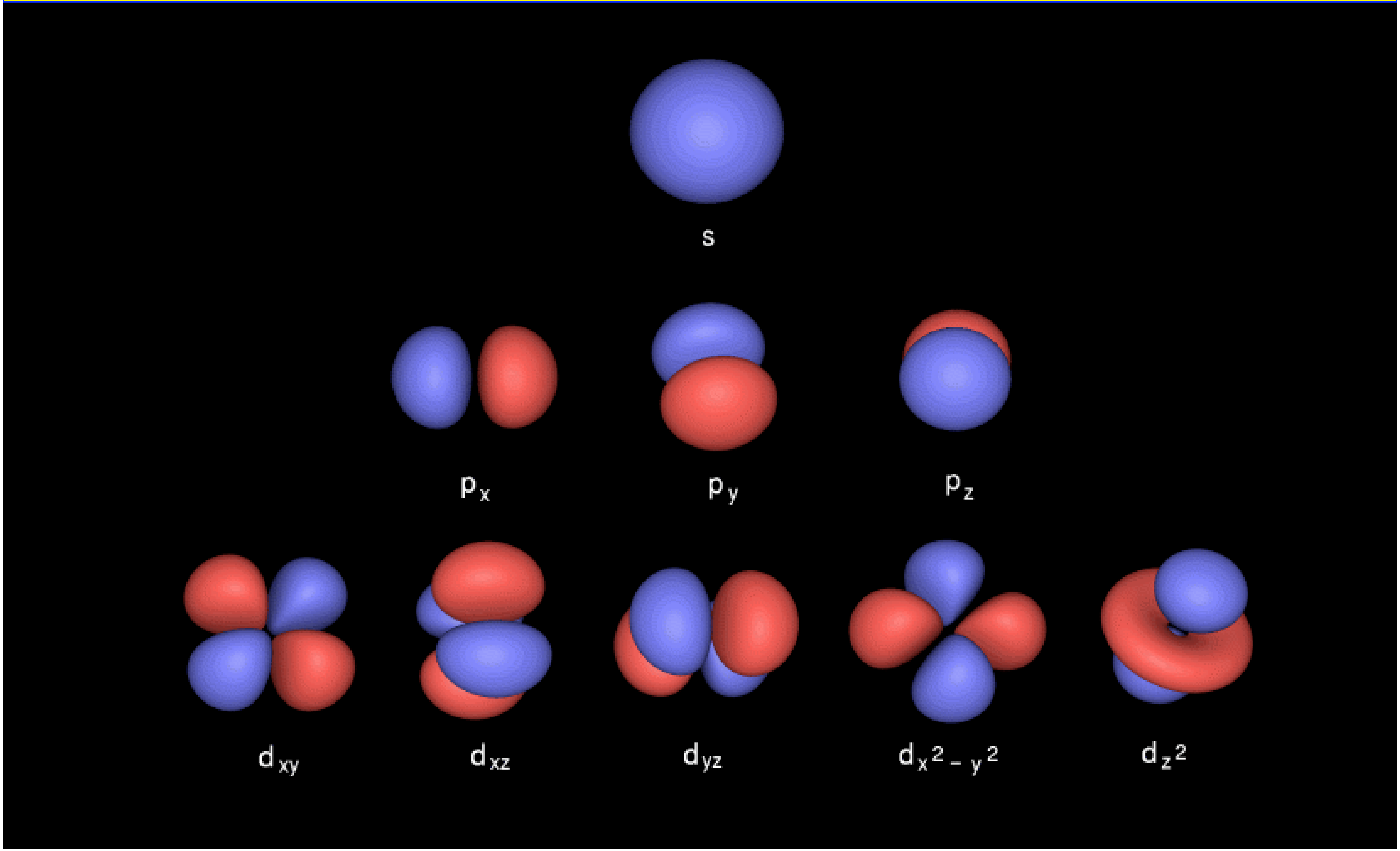
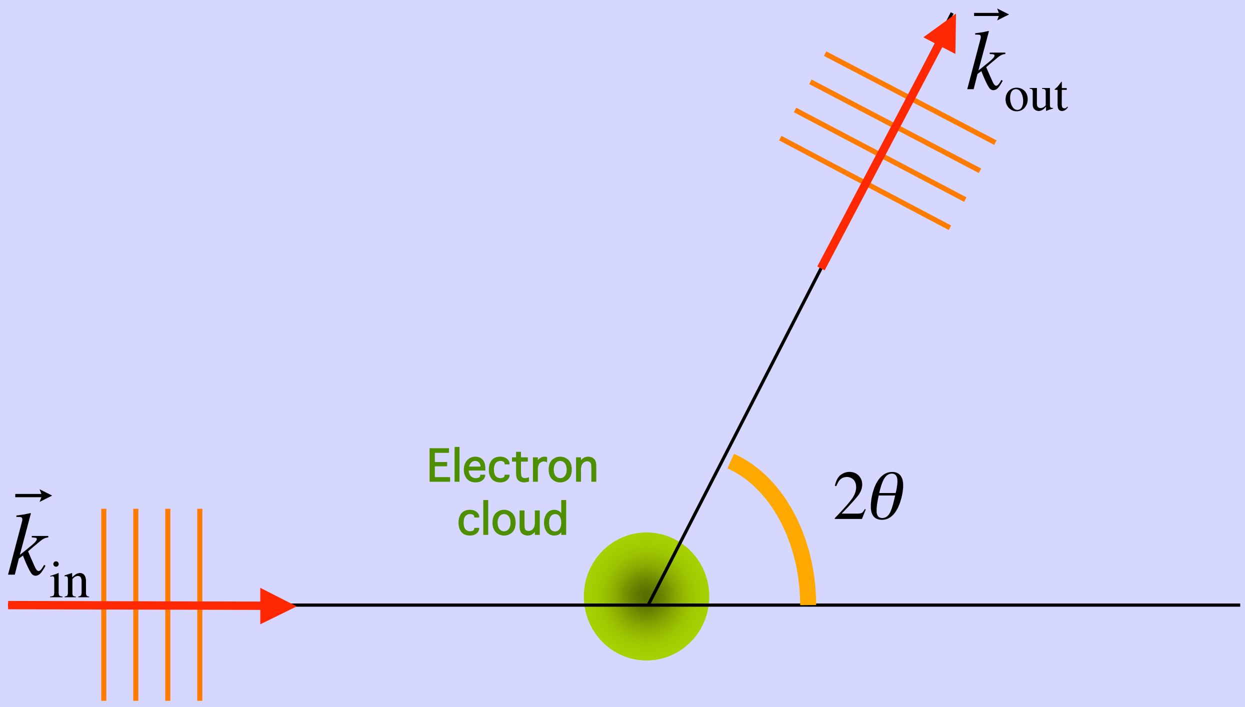
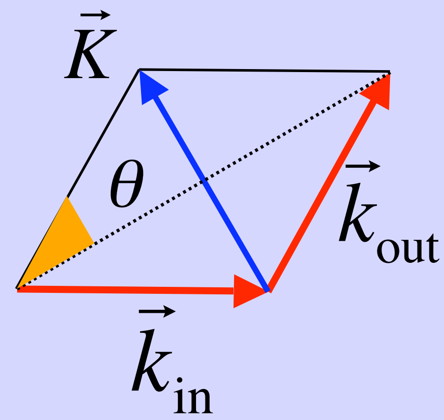


Interference effects





$$\left|\vec{K}\right| = 4\pi \frac{\sin\theta}{\lambda}$$

$$A(\vec{K}) = A_{\text{el}} \int \rho_e(\vec{r})\, e^{i\vec{K}\cdot\vec{r}}\, dV$$

Interference depends on wavelength

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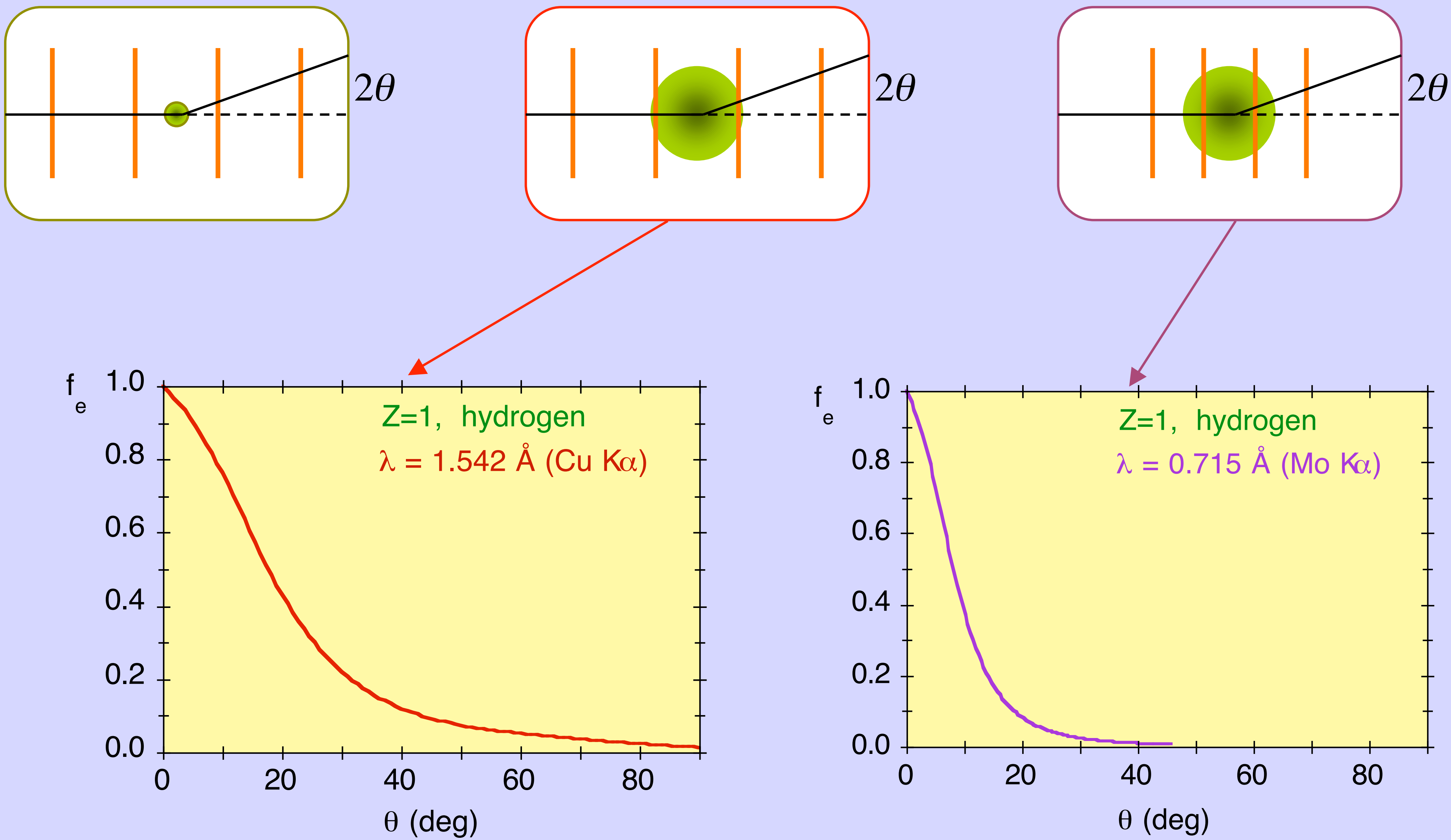

Electron scattering factor

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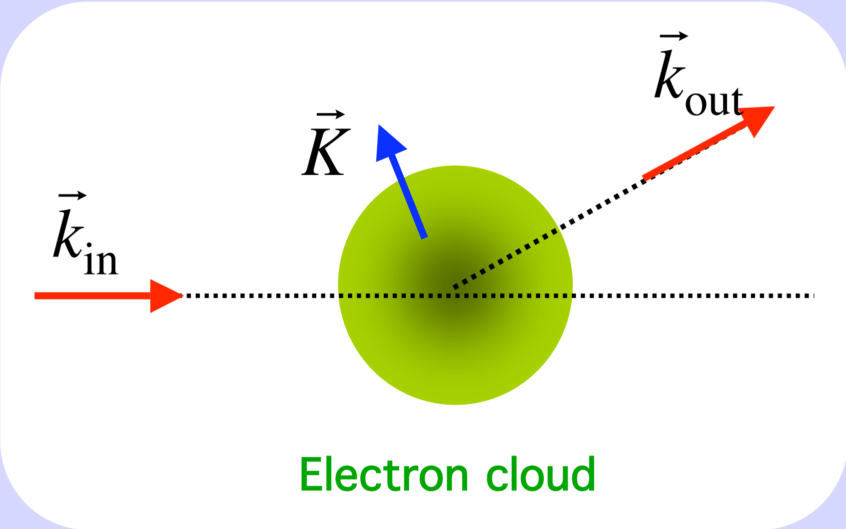

$$A(\vec{K}) = A_{el} \int \rho_e(\vec{r})\, e^{i\vec{K}\cdot\vec{r}}\, dV$$

$$f_e(\vec{K})$$

$$I(\vec{K}) = \left|A(\vec{K})\right|^2 = \left|A_{el}\right|^2 \left|f_e(\vec{K})\right|^2$$

Electronic units

$$A_{e.u.}(\vec{K}) = \frac{A(\vec{K})}{A_{el}} = f_e(\vec{K})$$

$$I_{e.u.}(\vec{K}) = \frac{\left|A(\vec{K})\right|^2}{\left|A_{el}\right|^2} = \left|f_e(\vec{K})\right|^2$$

Hydrogen scattering factor

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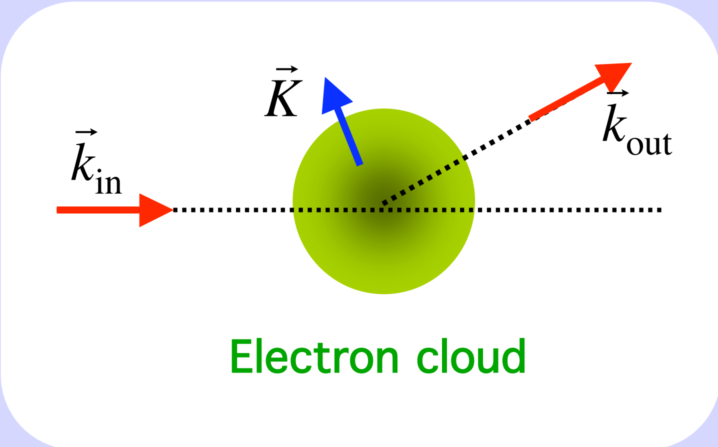

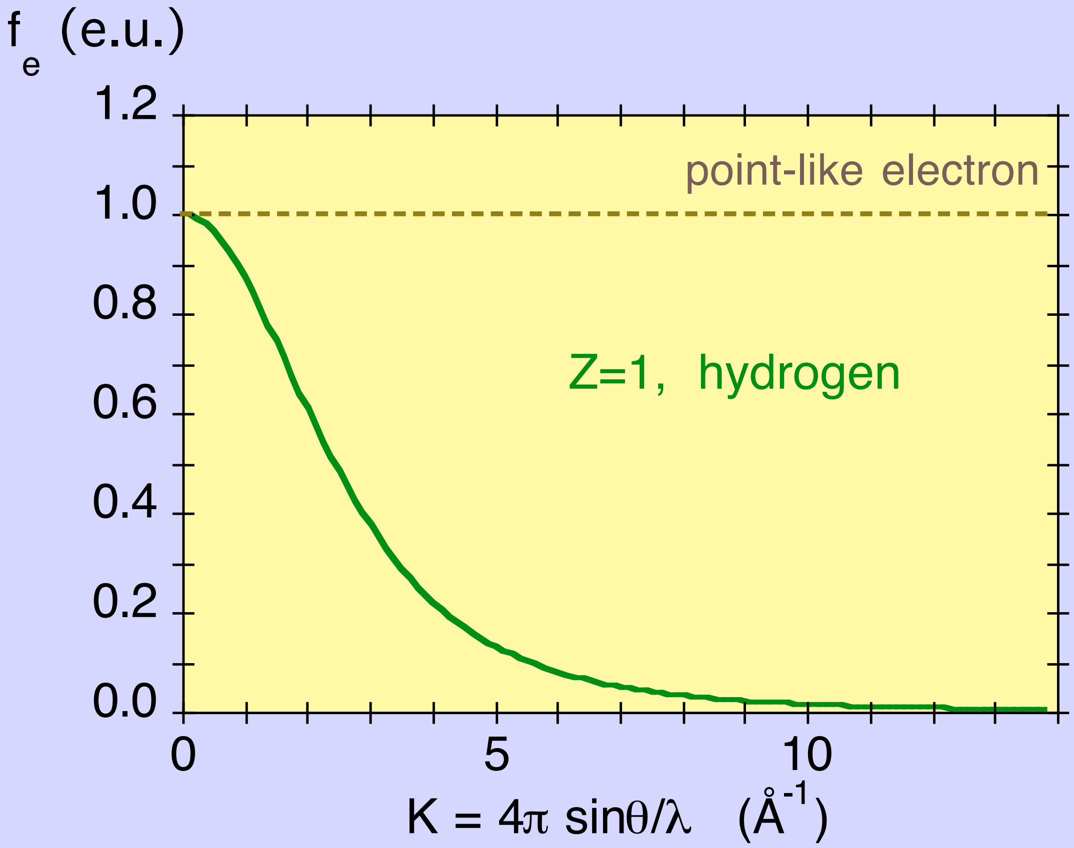

$$f_e(\vec{K}) = \int \rho_e(\vec{r})\, e^{i\vec{K}\cdot\vec{r}}\, dV$$

$$= 4\pi \int_0^\infty r^2 \rho_e(r) \frac{\sin Kr}{Kr}\, dr$$

for spherical symmetry

Atomic scattering factor (a)

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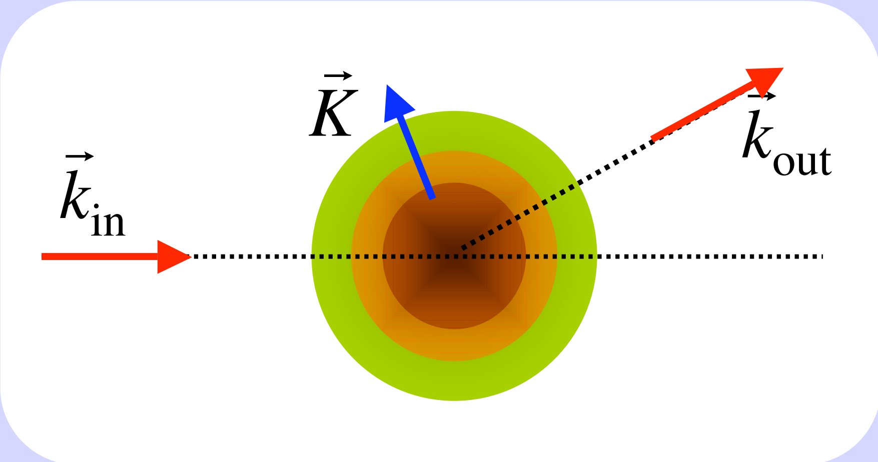

sum over electrons

$$f_0(\vec{K}) = \sum_n f_{e,n}(\vec{K})$$

$$= \sum_n \int \rho_{e,n}(\vec{r}) e^{i\vec{K}\cdot\vec{r}} dV$$

$$= \int \rho(\vec{r}) e^{i\vec{K}\cdot\vec{r}} dV$$

$$= 4\pi \int_0^\infty r^2 \rho_e(r) \frac{\sin Kr}{Kr} dr$$

total electronic density

for spherical symmetry

$$A_{e.u.}(\vec{K}) = f_0(\vec{K})$$

$$I_{e.u.}(\vec{K}) = \left| f_0(\vec{K}) \right|^2$$

Atomic scattering factor (b)

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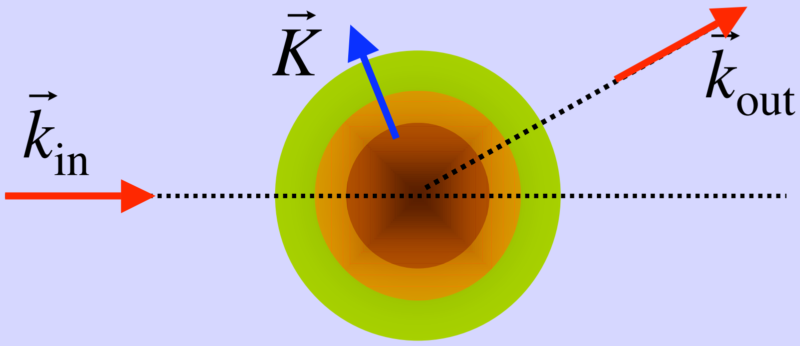
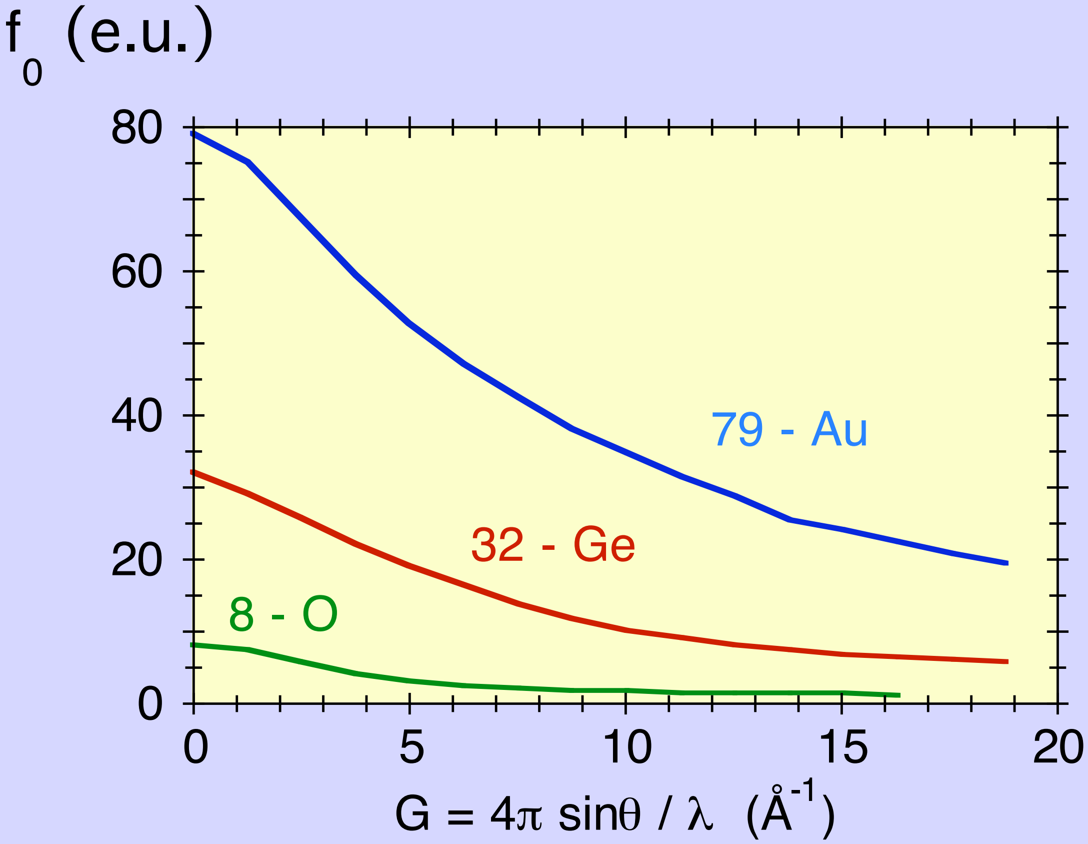


$$f_0\left(\vec{K}\right) = \int \rho(\vec{r})\, e^{i\vec{K}\cdot\vec{r}} dV$$



- Deviations from classical treatment
  b) Compton effect

Compton experiment (1922)

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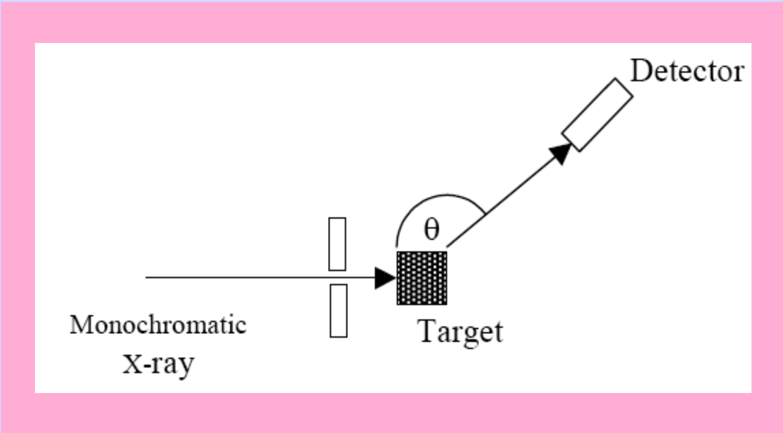

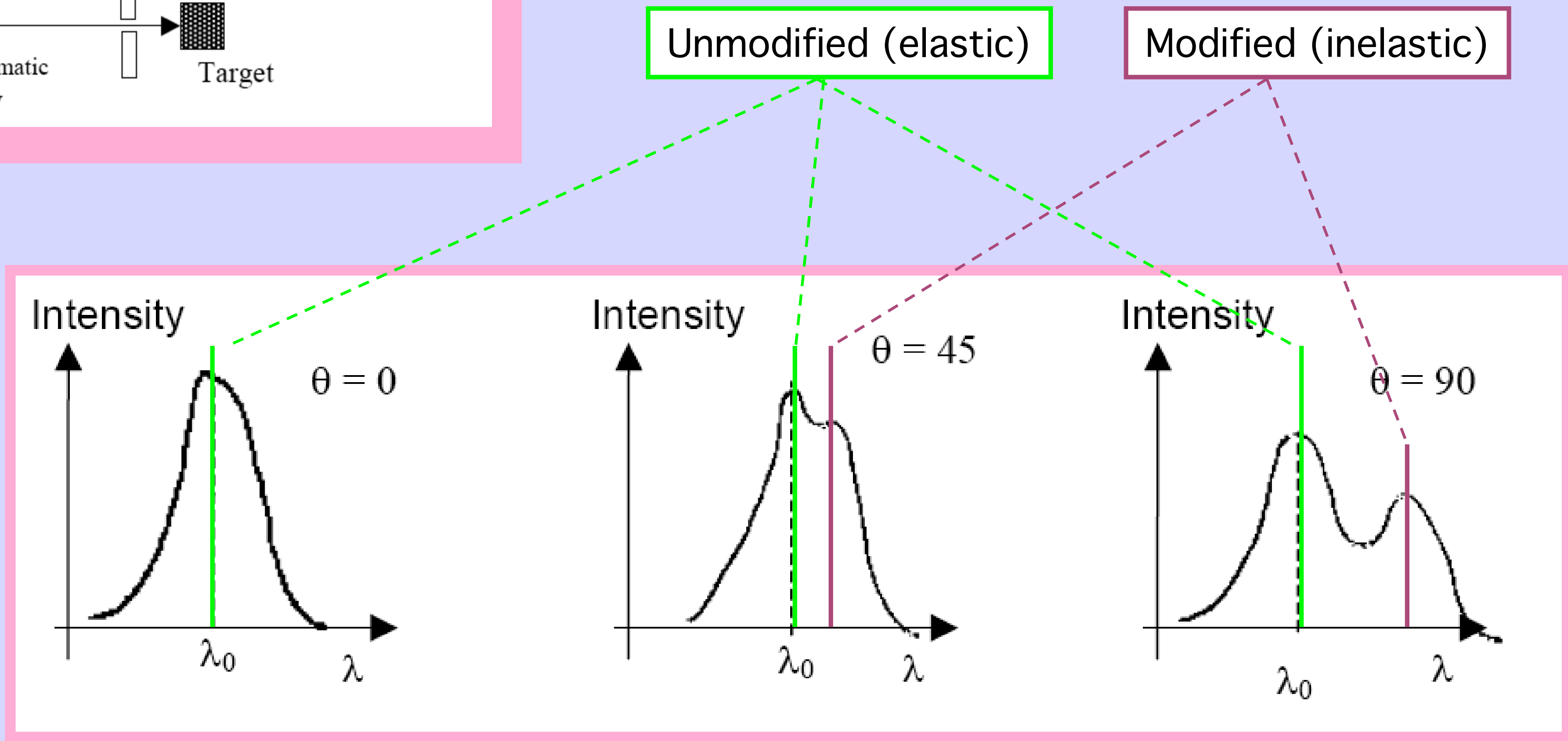

Compton effect

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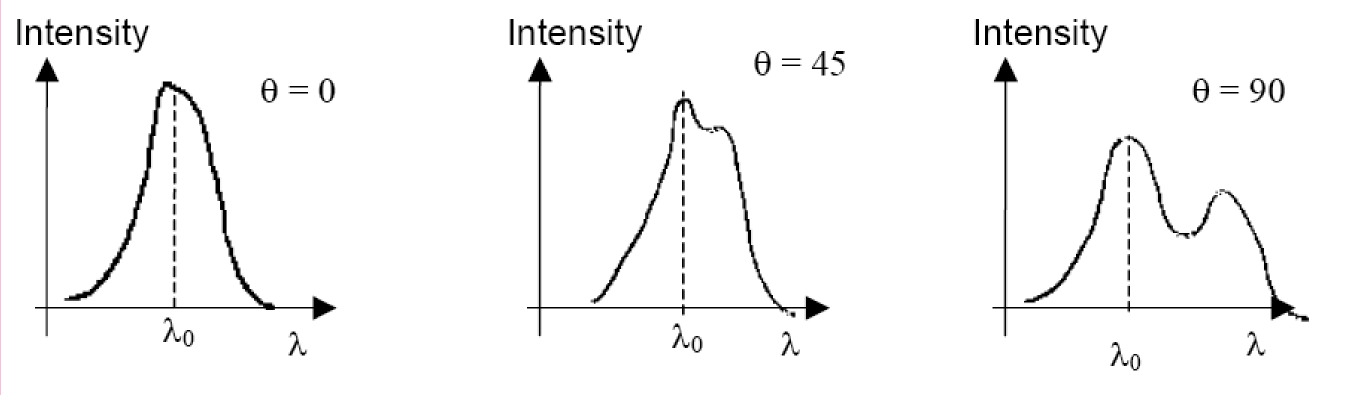

$$\lambda' = \lambda + \frac{\hbar}{mc}(1-\cos\theta)$$

$$= \lambda + \lambda_c(1-\cos\theta)$$

0.002426 Å
(Compton wavelength)

$$E'[keV] = \frac{E[keV]}{1+0.001957\,E\,(1-\cos\theta)}$$

Modified scattering - 1 electron

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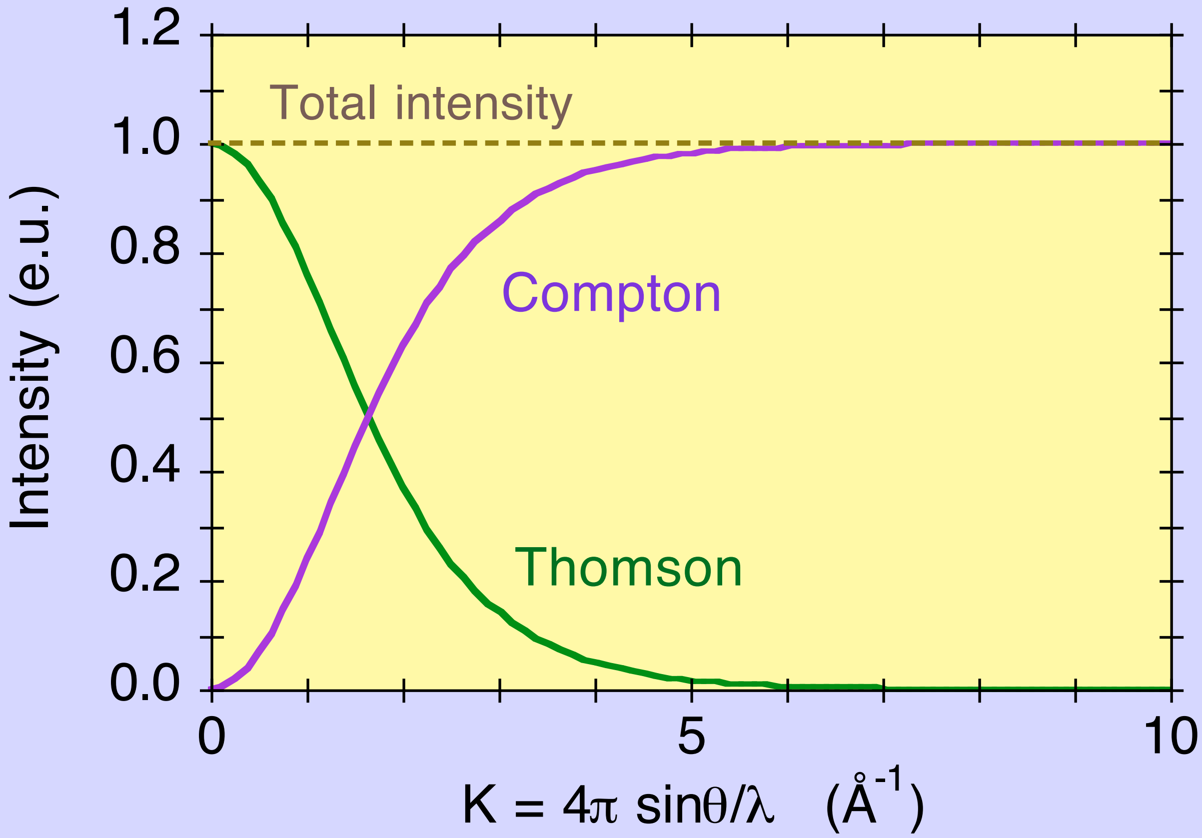
$$I_{e,\mathrm{unmod}} + I_{e,\mathrm{mod}} = 1$$

$$f_e^2 + I_{e,\mathrm{mod}} = 1$$

$$I_{e,\mathrm{mod}} = 1 - f_e^2$$

Hydrogen



Modified scattering - atoms

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Thomson

$$I_{\text{un mod}} = \left| \sum_{n=1}^{Z} f_{e,n} \right|^2$$

$\leq Z^2$

coherent scattering
(sum of amplitudes)

Compton

$$I_{\text{mod}} = \sum \left(I_{e,\text{mod}} \right)_n$$
$$= \sum (1 - f_{e,n}^2)$$
$$= Z - \sum_{n=1}^{Z} f_{e,n}^2$$

$\leq Z$

uncoherent scattering
(sum of intensities)

Thomson .vs. Compton

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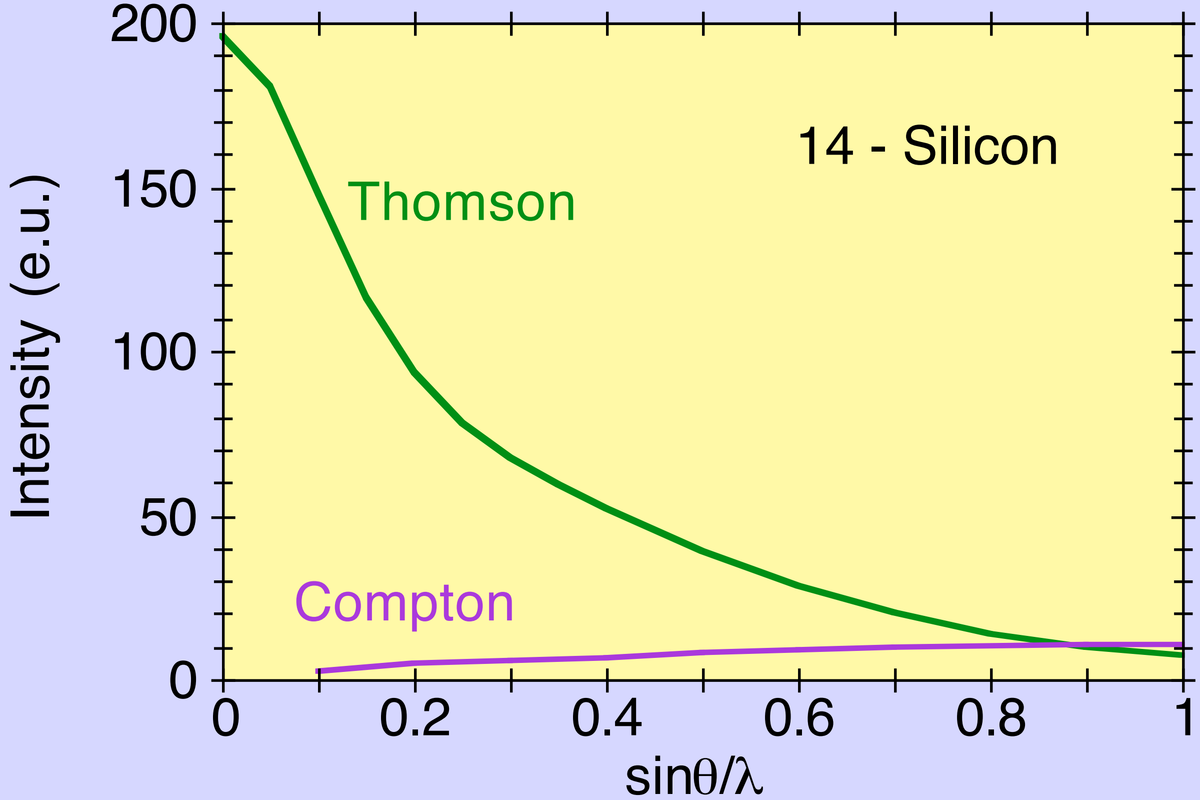
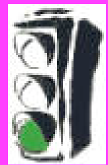
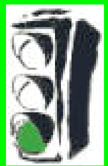
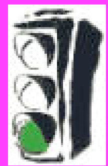


Scattering of X-rays from an electron

Thomson scattering		Compton scattering
$\nu' = \nu$		$\nu' \neq \nu$
Elastic (“coherent”, “unmodified”)		**Inelastic** (“incoherent”, “modified”)
 Classical treatment	⟵ Free electron ⟶	Quantum treatment 
 Tightly bound	⟵ Electron in atom ⟶	Loosely bound 

- Deviations from classical treatment
  c) Electron bonding

Effect of electron bonding (a)

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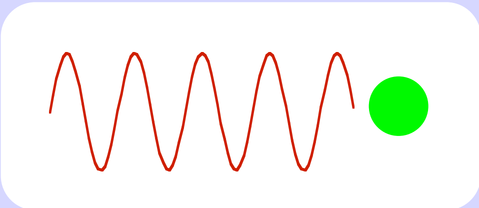

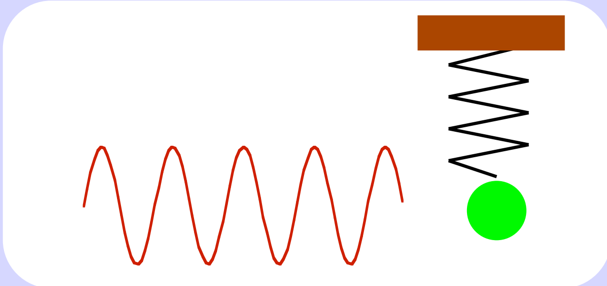

Free electron

$$\frac{d^2\vec{r}}{dt^2} = -\frac{e}{m}\vec{E}_0 e^{i\omega t}$$

E field

Bound electron

$$\frac{d^2\vec{r}}{dt^2} + \gamma\frac{d\vec{r}}{dt} + \omega_0^2\vec{r} = -\frac{e}{m}\vec{E}_0 e^{i\omega t}$$

Complex notation

Damping force

Elastic force

E field

Damped oscillator model

Model of bound electron

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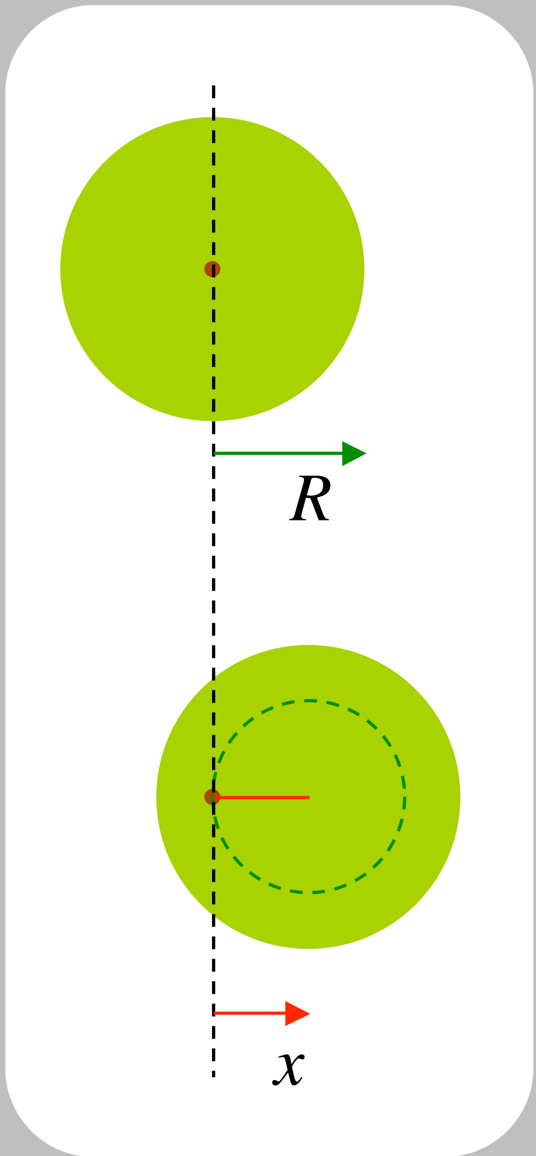

Spherical distribution of negative charge centered on the positive nucleus

Displacement x

Restoring force

$$|F| = \frac{1}{4\pi\varepsilon_0}\frac{Qq_x}{x^2} \approx \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{x^2}\frac{x^3}{R^3}$$

$$|F| \propto x$$

Effect of electron bonding (b)

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Free electron

$$\frac{d^2\vec{r}}{dt^2} = -\frac{e}{m}\vec{E}_0 e^{i\omega t}$$

Bound electron:

$$\frac{d^2\vec{r}}{dt^2} + \gamma\frac{d\vec{r}}{dt} + \omega_0^2\vec{r} = -\frac{e}{m}\vec{E}_0 e^{i\omega t}$$

Oscillating solution

$$\vec{r}(t) = \vec{r}_0 \, e^{i\omega t}$$

Complex notation

Amplitude of oscillation

Free electron:

$$\vec{r}_0 = \frac{e\vec{E}_0}{m}\frac{1}{\omega^2}$$

Bound electron:

$$\vec{r}_0 = \frac{e\vec{E}_0}{m}\frac{1}{\omega^2 - \omega_0^2 - i\gamma\omega}$$

Effect of electron bonding (c)

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Free electron

$$\vec{r}_0 = \frac{e\vec{E}_0}{m}\frac{1}{\omega^2}$$

Amplitude of oscillation

Bound electron:

$$\vec{r}_0 = \frac{e\vec{E}_0}{m}\frac{1}{\omega^2 - \omega_0^2 - i\gamma\omega}$$

Complex notation

spatial distribution

$$A(\vec{K}) = A_{\mathrm{el}}\ f_e(\vec{K})$$

Amplitude of scattering

$$A\left(\vec{K}\right) = A_{\mathrm{el}}\ f_e(\vec{K})\ \frac{\omega^2}{\omega^2 - \omega_0^2 - i\gamma\omega}$$

Scattering factor

Anomalous scattering factor

"Anomalous" terms (1 electron)

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$$A\left(\vec{K}\right) = E_{\text{free}}^{\text{out}}\ f_e(\vec{K}) \left[\frac{\omega^2}{\omega^2 - \omega_0^2 - i\gamma\omega} \right]$$

$$= E_{\text{free}}^{\text{out}} \left[f_e(\vec{K}) \frac{\omega^2\left(\omega^2 - \omega_0^2\right)}{\left(\omega^2 - \omega_0^2\right)^2 + \gamma^2\omega^2} \quad + \quad i\ f_e(\vec{K}) \frac{\gamma\omega^3}{\left(\omega^2 - \omega_0^2\right)^2 + \gamma^2\omega^2} \right]$$

$$= E_{\text{free}}^{\text{out}} \left[f_e(\vec{K}) + f_e(\vec{K}) \frac{\omega_0^2\left(\omega^2 - \omega_0^2\right) - \gamma\omega^2}{\left(\omega^2 - \omega_0^2\right)^2 + \gamma^2\omega^2} \quad + \quad i\ f_e(\vec{K}) \frac{\gamma\omega^3}{\left(\omega^2 - \omega_0^2\right)^2 + \gamma^2\omega^2} \right]$$

$$A\left(\vec{K}\right) = A_{\text{el}} \left[f_e(\vec{K}) + f'_e + i f''_e \right]$$

"Anomalous" terms

“Anomalous” scattering factor (atoms)

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Real part

$$f\left(\vec{K}\right) = f_0\left(\vec{K}\right) + f_1 + i\, f_2$$

anomalous terms
(~ indep. from K)

Imaginary part

Absorption

(for forward scattering)

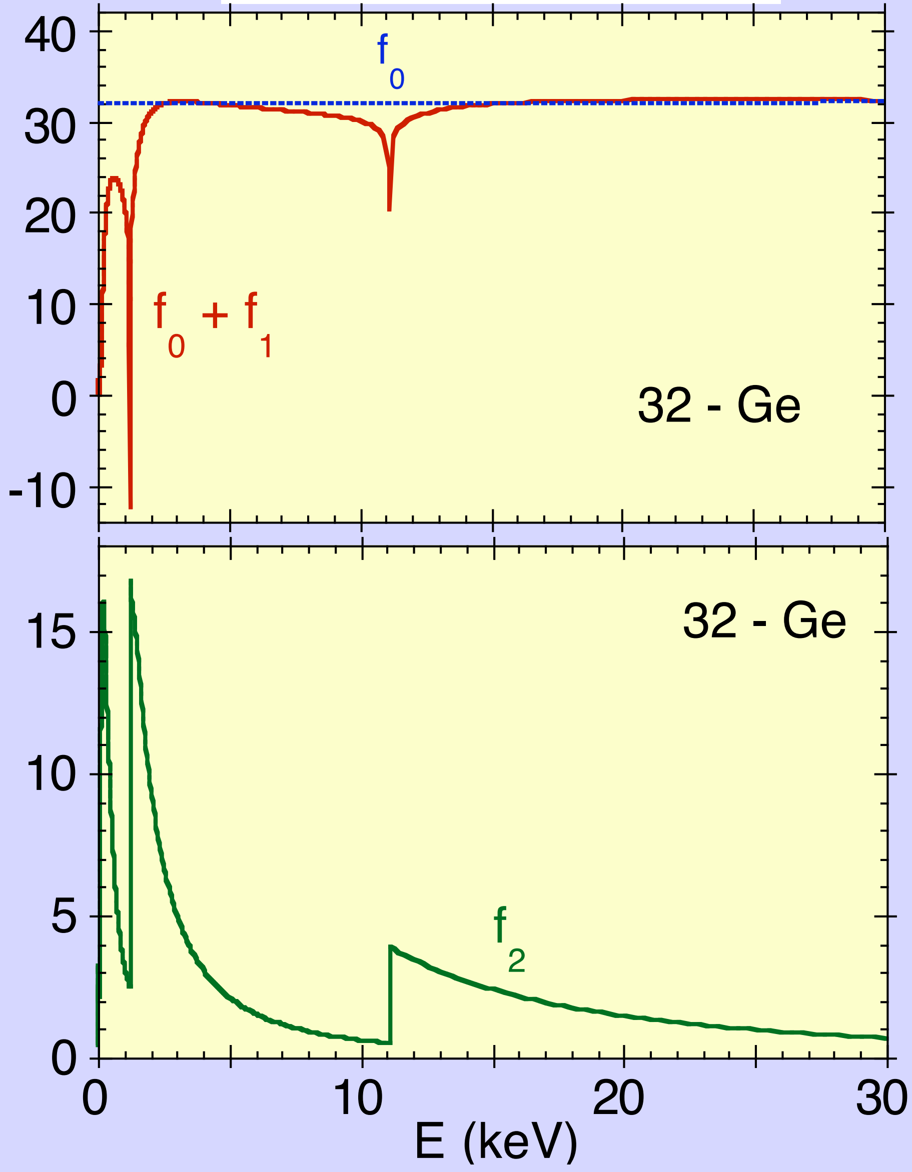

Photo-electric absorption
a) Phenomenology

Attention of X-rays

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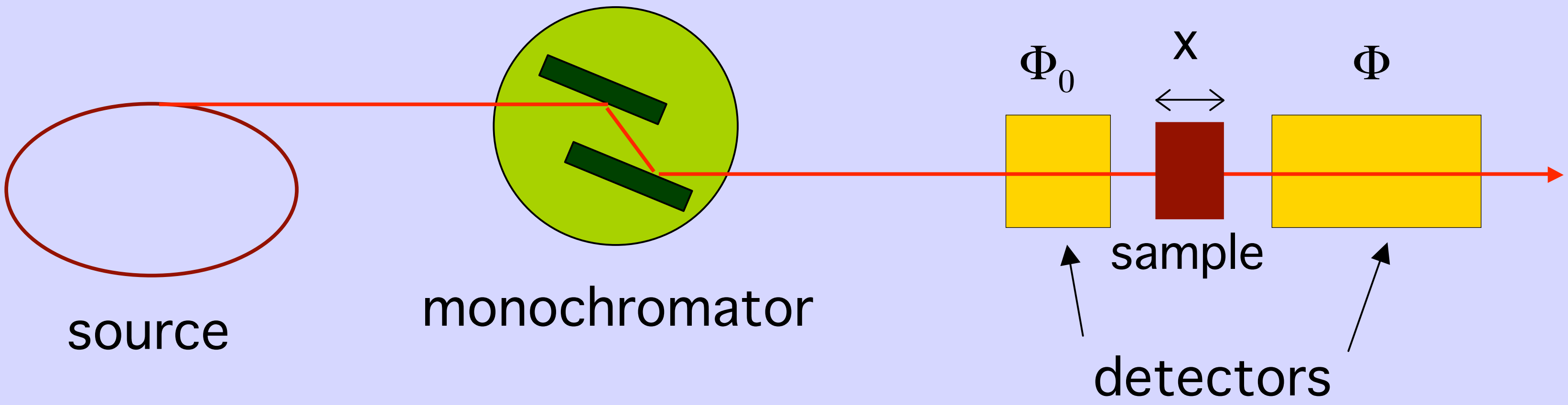

Exponential attenuation

$$\Phi = \Phi_0 \exp\left[-\mu(\omega)\, x\right]$$

Attenuation coefficient

$$\mu(\omega) = \frac{1}{x} \ln \frac{\Phi_0}{\Phi}$$

Atomic cross sections

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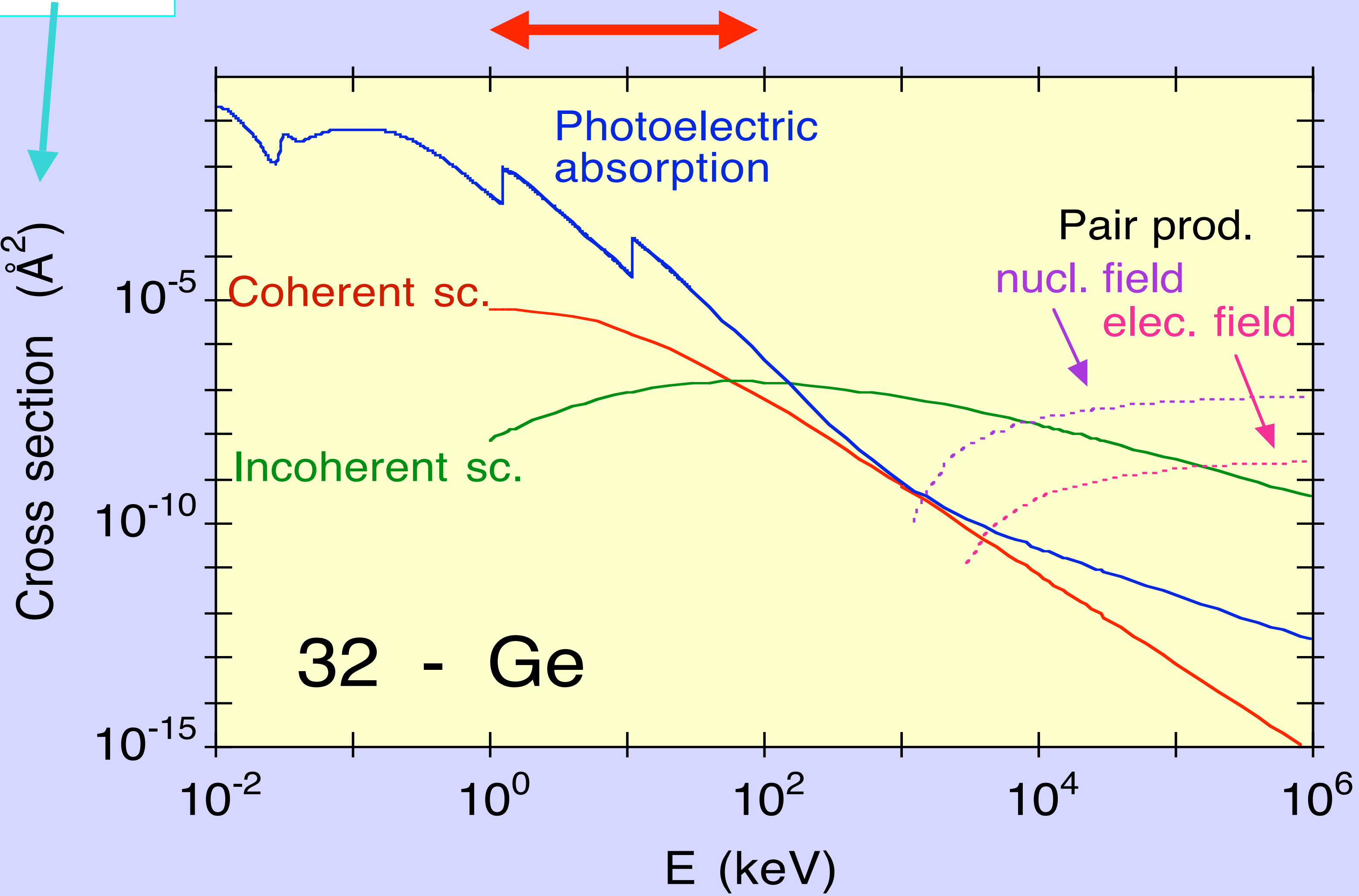
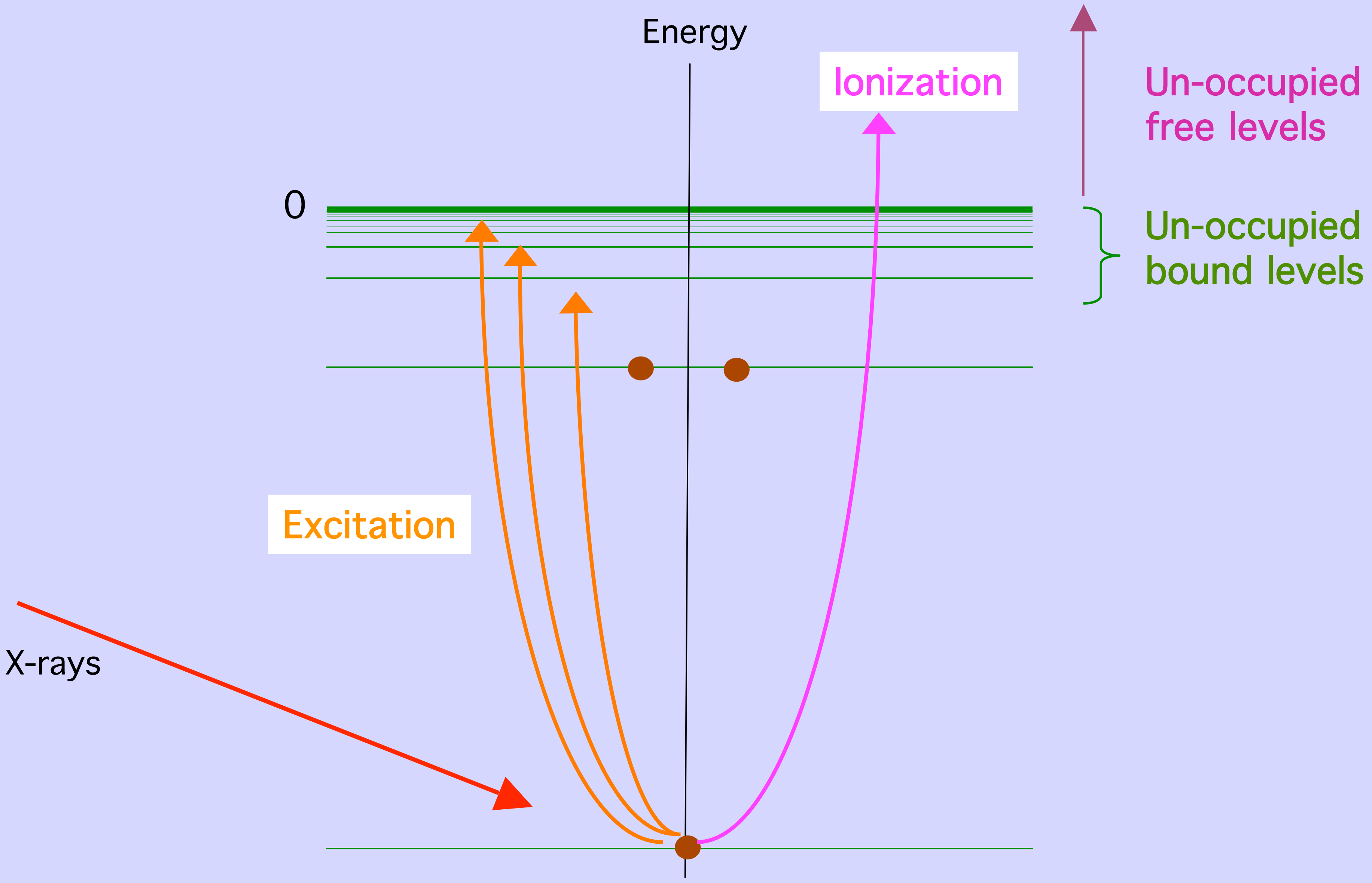
$$\mu(\omega) = \frac{N_a \rho}{A} \mu_a(\omega)$$



Excitation and ionization



Photoelectric absorption (a)

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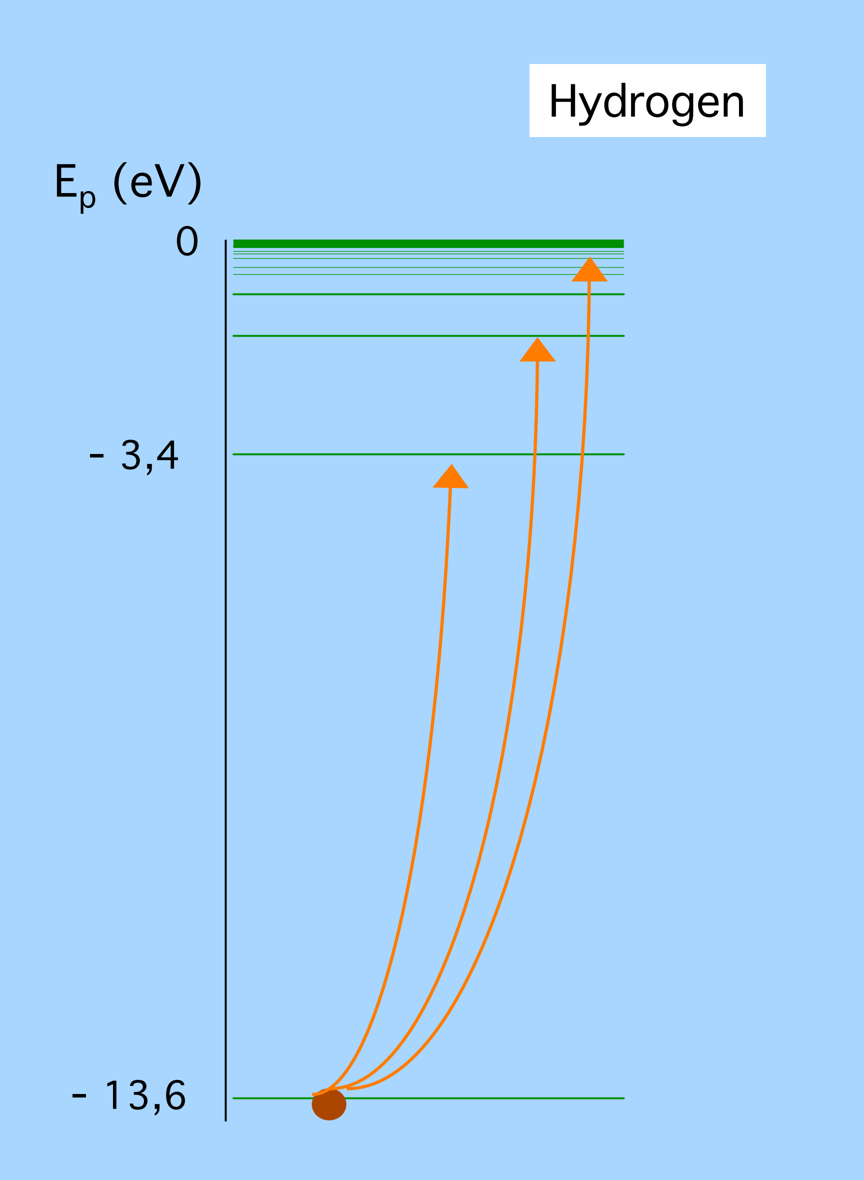

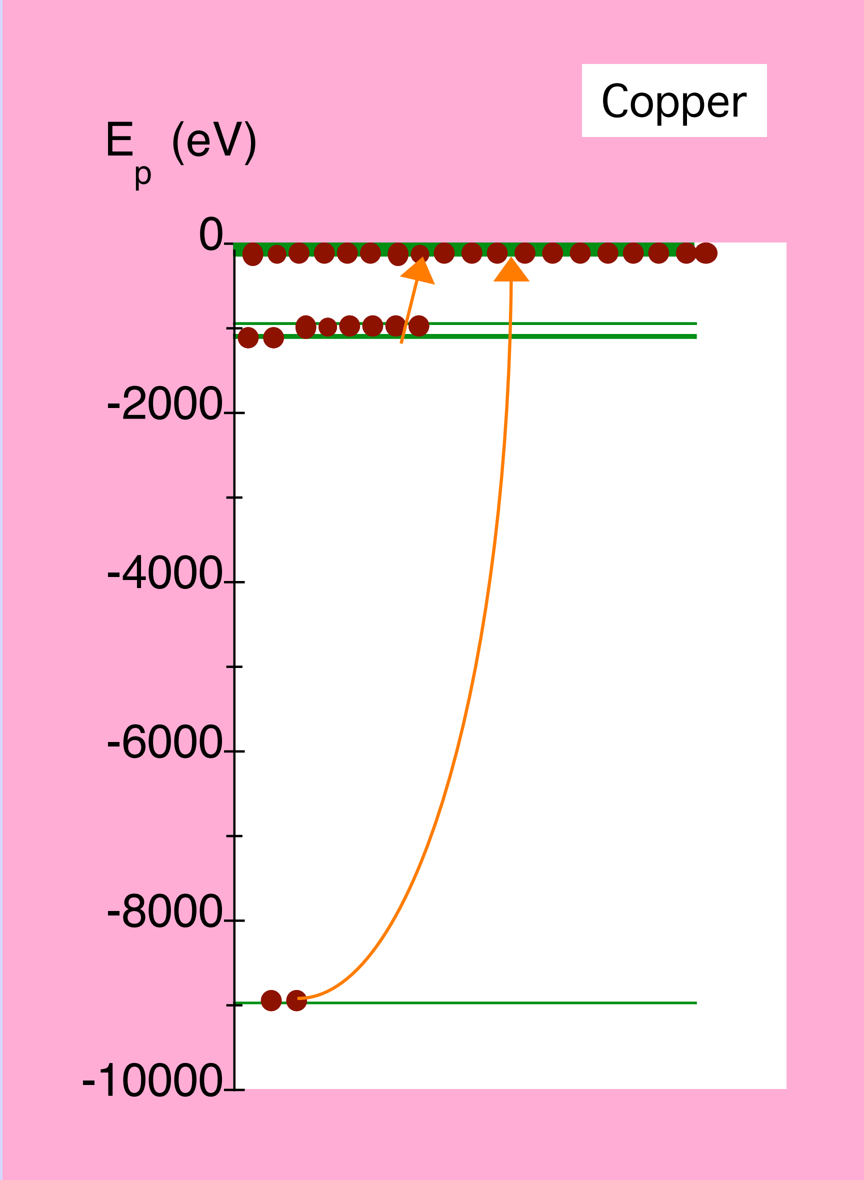

X-ray absorption spectroscopy

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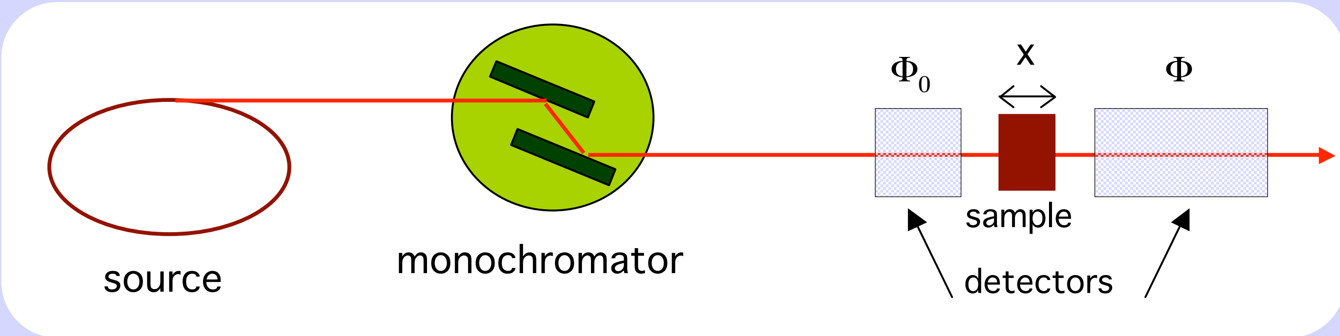
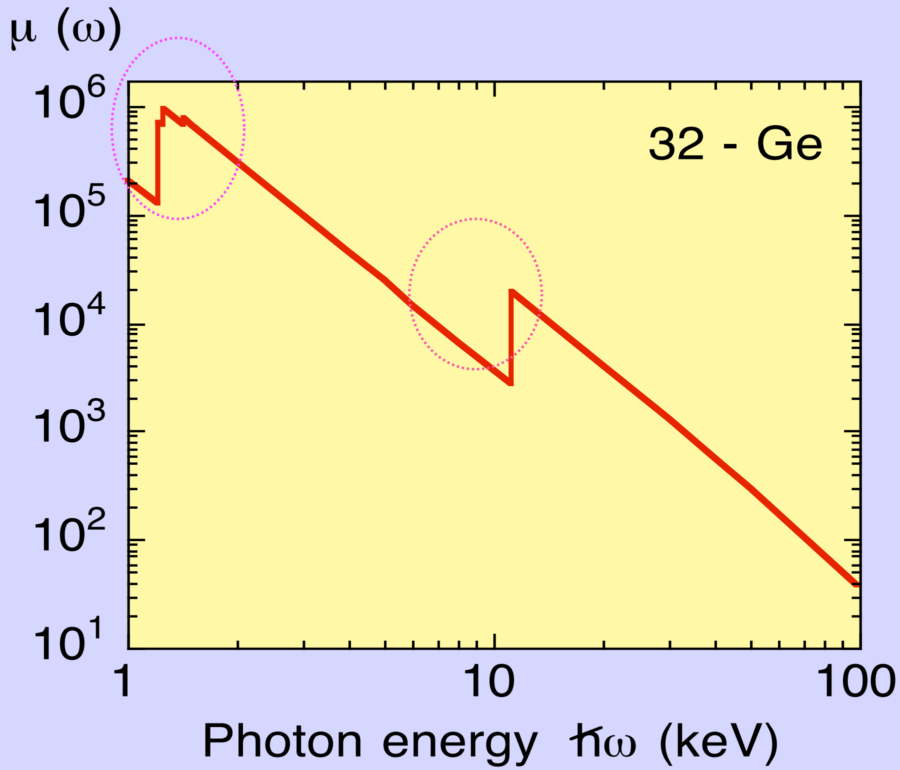


- Exponential attenuation

$$\Phi = \Phi_0 \exp\left[-\mu(\omega)\, x\right]$$

- Attenuation coefficient

$$\mu(\omega) = \frac{1}{x} \ln \frac{\Phi_0}{\Phi}$$



$$\mu(\omega) \propto \frac{Z^4}{(\hbar\omega)^3}$$

+

Edges

X-ray absorption edges

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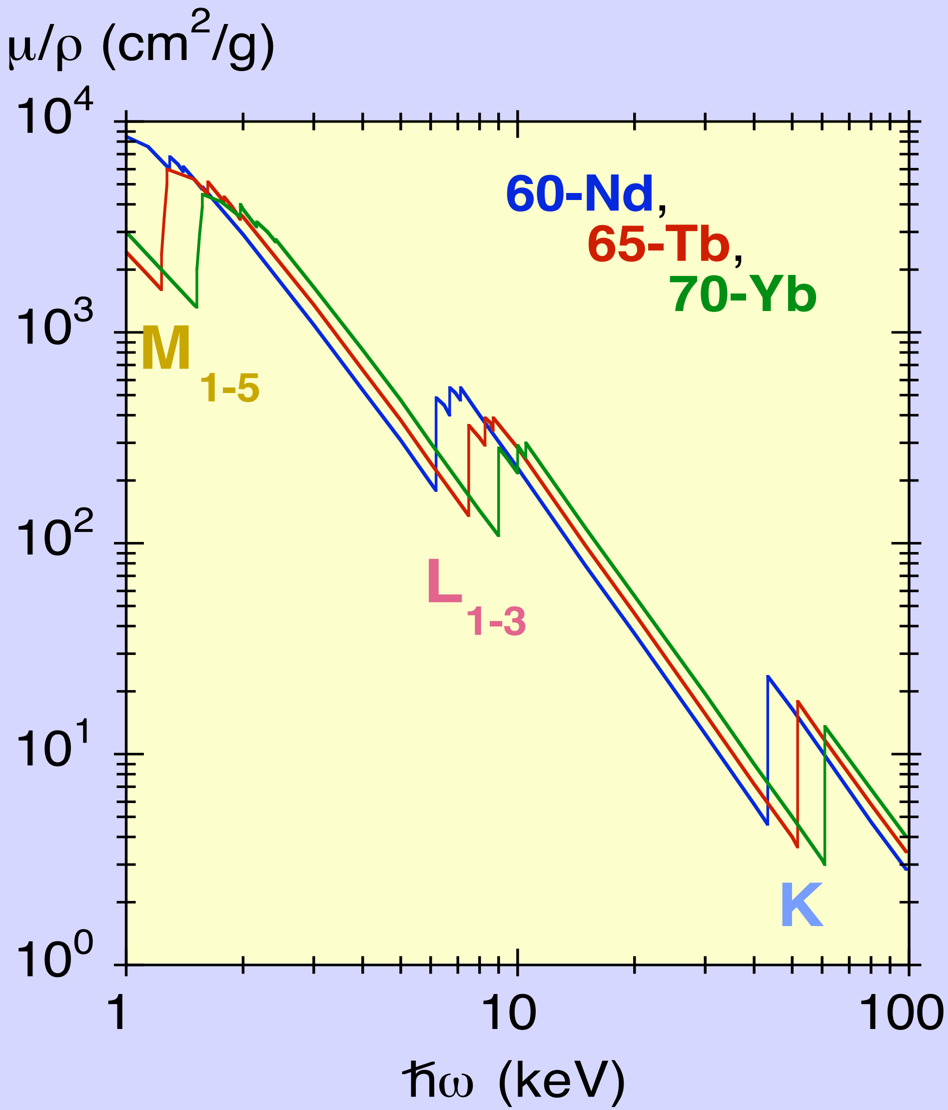
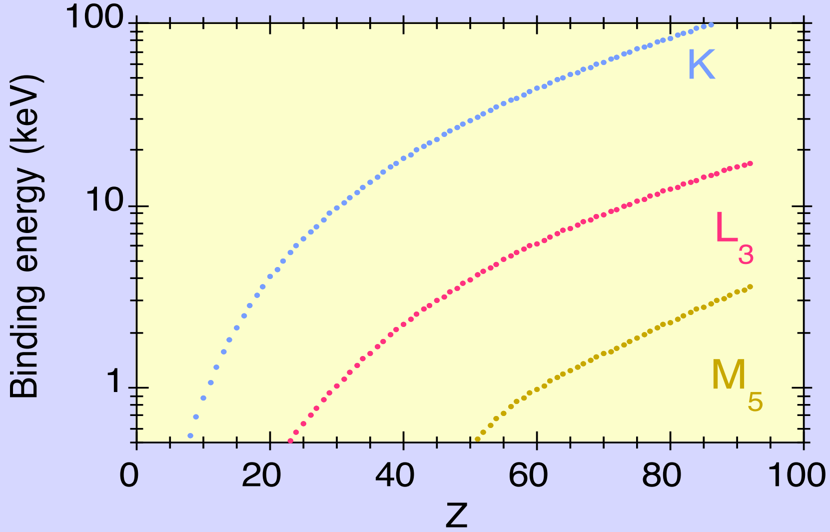


K	1s	
L_1	2s	
L_2	$2p_{1/2}$	
L_3	$2p_{3/2}$	Z > 9
M_1	3s	
M_2	$3p_{1/2}$	
M_3	$3p_{3/2}$	
M_4	$3d_{3/2}$	
M_5	$3d_{5/2}$	Z > 29

. . . .



Fine Structure: Atoms

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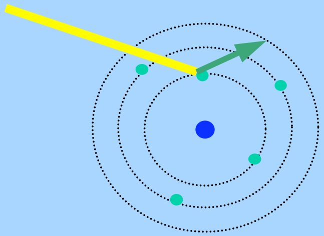
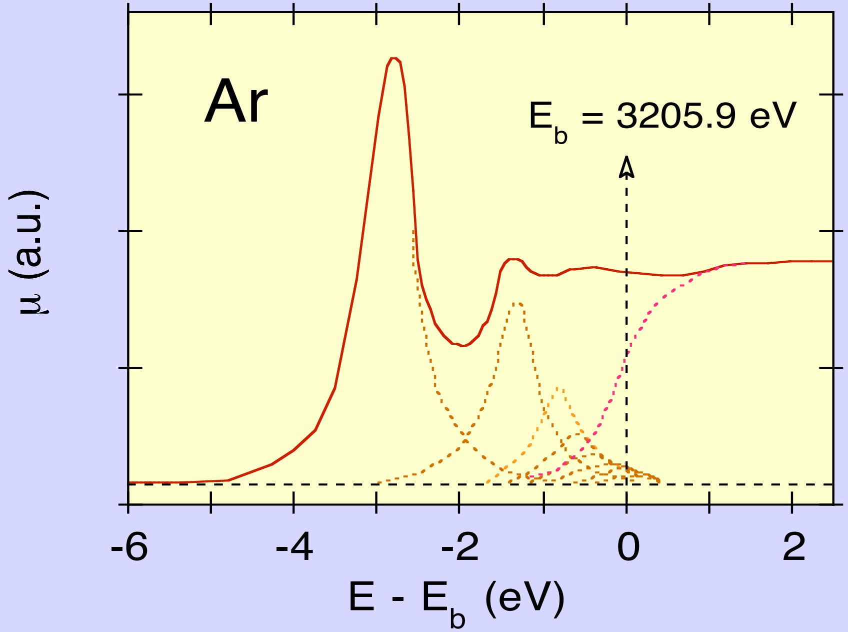
hν < binding energy

Core electron → unoccupied levels

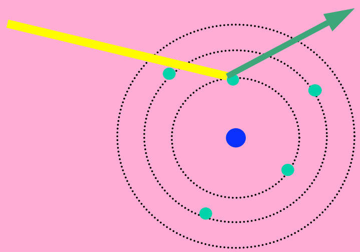


⇒ Edge Fine Structure

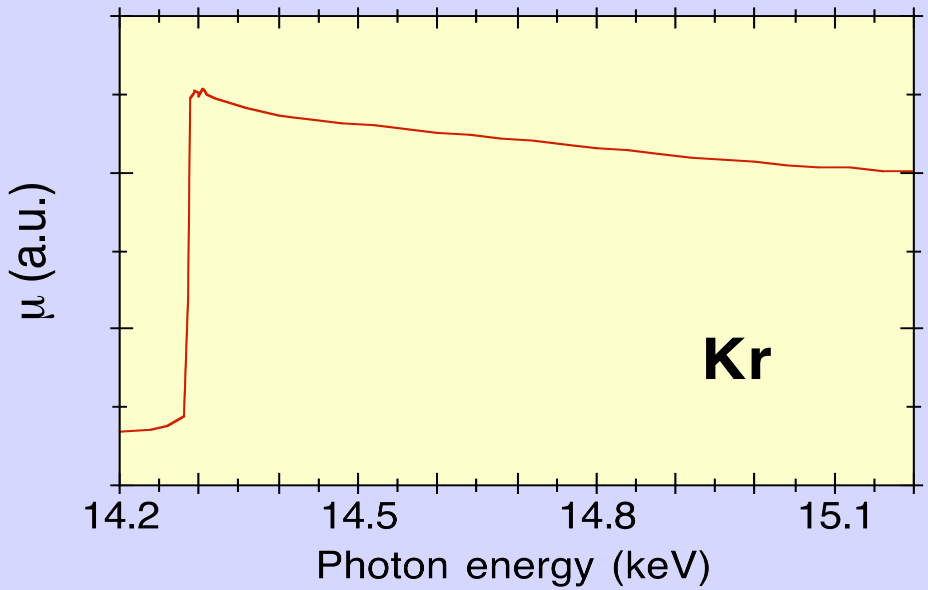


hν > binding energy

Core electron → continuum



⇒ Smooth μ



Fine Structure: Molecules and Condensed systems

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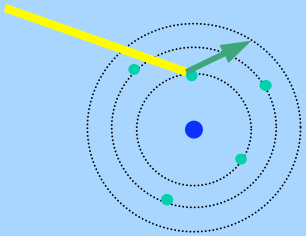
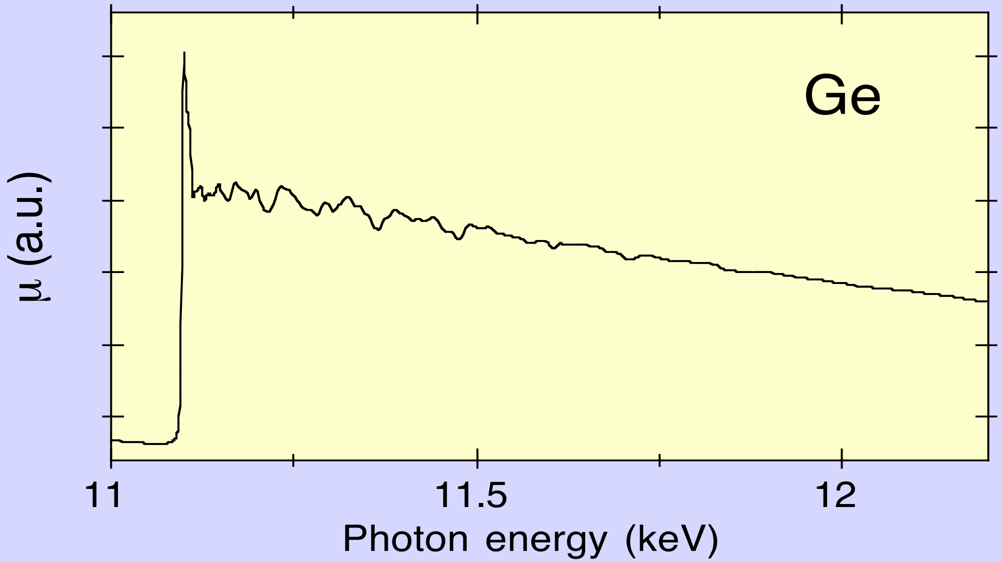
hν ≈ binding energy

Core electron → unoccupied levels



⇒ Edge Fine Structure



hν > binding energy

Core electron → continuum → Outgoing wave-function

Back-scattering from neighbouring atoms → Incoming wave-function

Outgoing wave-function ⇕ Incoming wave-function: INTERFERENCE ⇒ Extended Fine Structure

XAFS: X-ray Absorption Fine Structure

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X-ray Absorption Near Edge Structure
Near Edge X-ray Absorption Fine Structure

Electronic transitions
Photo-electron multiple scattering

Extended X-ray Absorption
Fine Structure

Mainly photo-electron single scattering

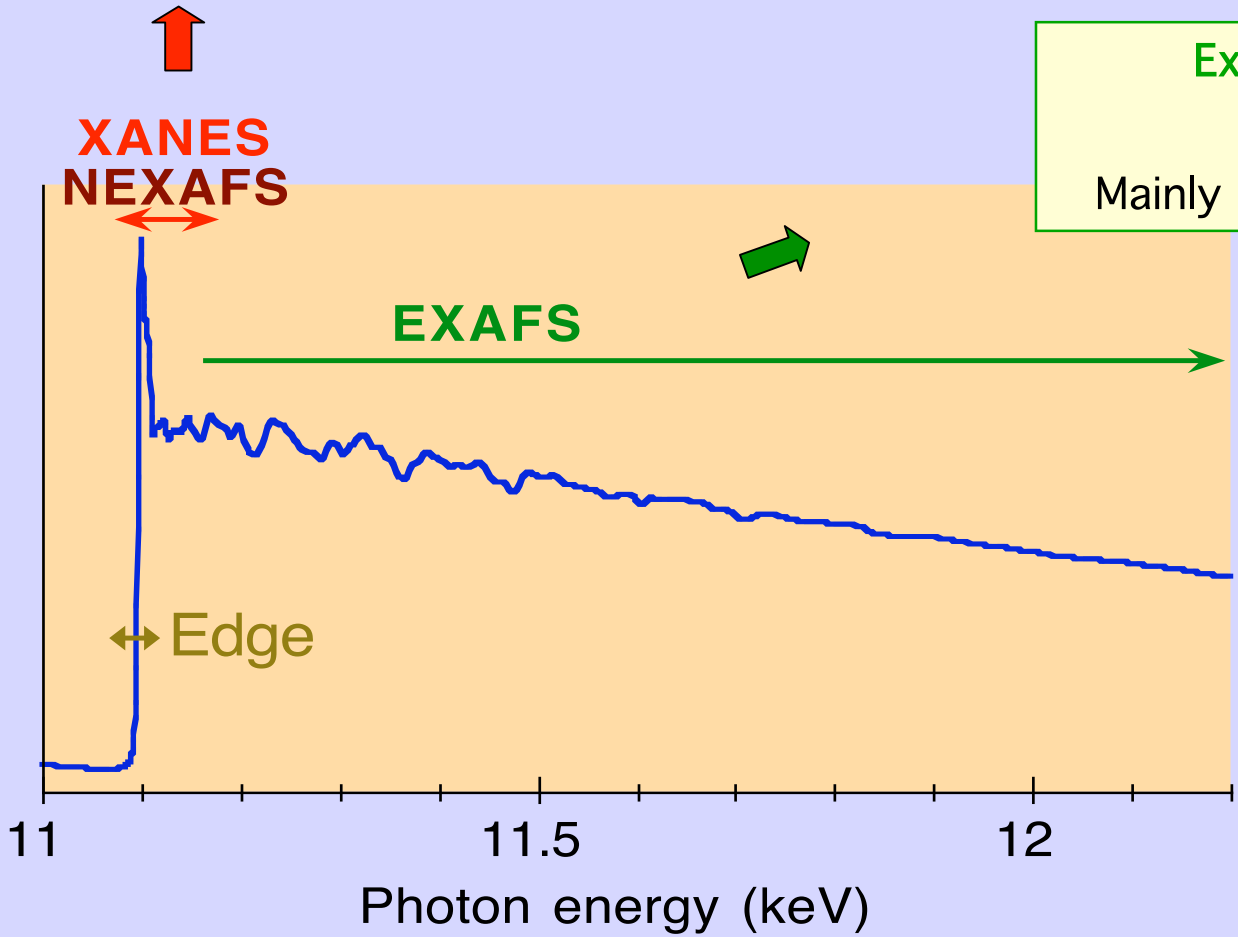

Photo-electric absorption

b) Theory

Transition probability W_{if}

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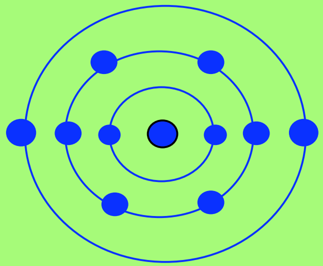
$$\mu(\omega) = \frac{2\hbar}{\varepsilon_0 \omega A_0^2}\, n \sum_f W_{if}$$

n = atomic density
A_0 = vector potential amplit.

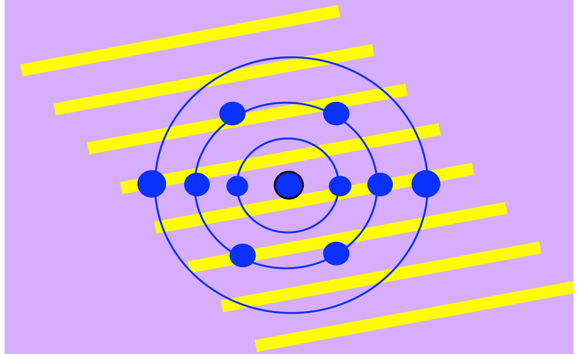
Initial atomic state



$|\Psi_i\rangle$

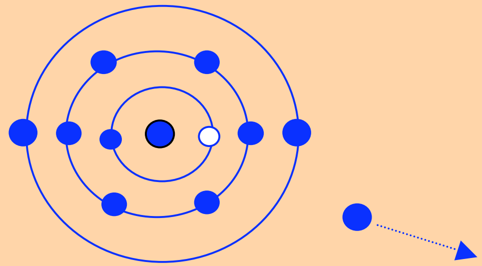
Stationary ground state

Interaction



$|\Psi(t)\rangle$

Final atomic state



$|\Psi_f\rangle$

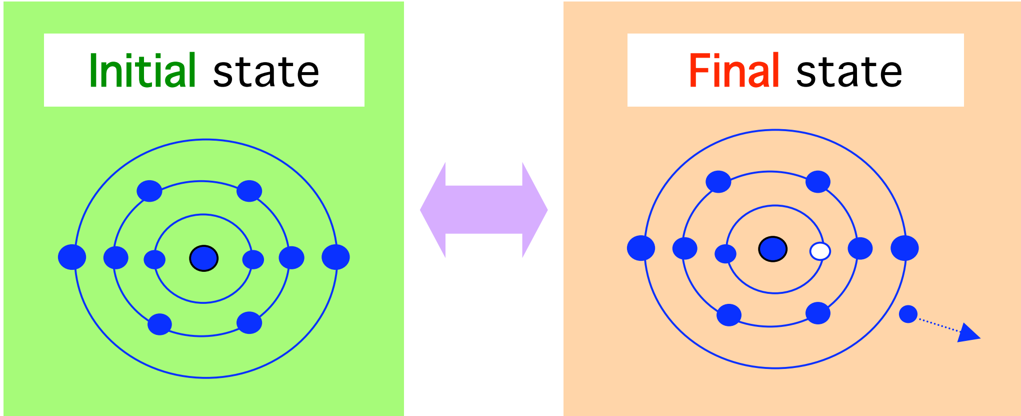
Stationary excited state

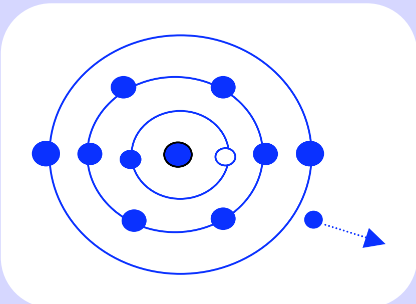
$W_{if} = ?$

Approximations

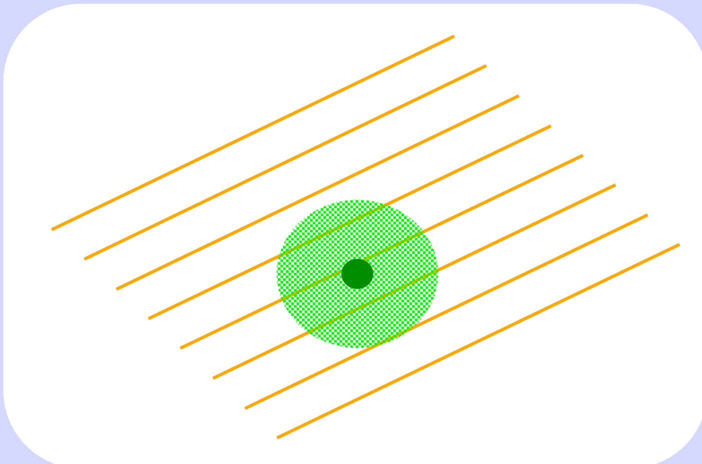
Weak interaction

Time-dependent perturbation theory
1st order





One electron



Electric dipole approximation

Absorption coefficient

1 active electron — passive electrons

$$\mu_{\mathrm{el}}(\omega) \propto \left|\langle \psi_i | \hat{\eta}\cdot\vec{r} | \psi_f \rangle\right|^2 \delta\left(E_i + \hbar\omega - E_f\right) \left|\langle \Psi_i^{N-1} | \Psi_f^{N-1} \rangle\right|^2$$

Energy conservation — Superposition integral

X-ray polarization

Electron position

$$\left|\langle \psi_i | \hat{\eta}\cdot\vec{r} | \psi_f \rangle\right|^2$$

Initial state

Final state

$$\left| \int \psi_i^*(\vec{r})\, \hat{\eta}\cdot\vec{r}\, \psi_f(\vec{r})\, d\vec{r} \right|^2$$

Electric dipole approximation

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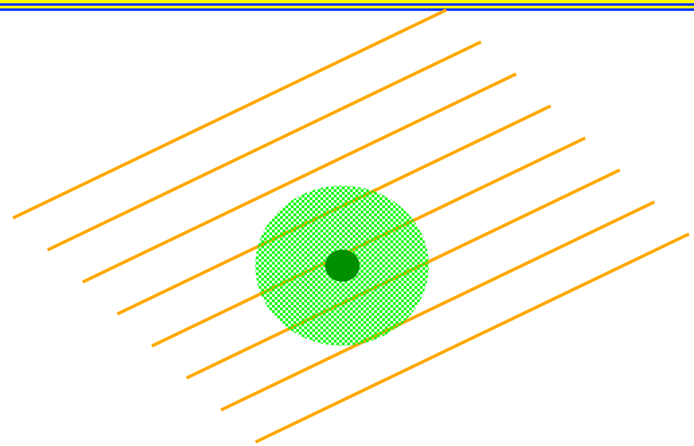

$$e^{i\vec{k}\cdot\vec{r}} = 1 + i\vec{k}\cdot\vec{r} - \ldots \approx 1$$

$$H_I \propto e^{i\vec{k}\cdot\vec{r}}\,\hat{\eta}\cdot\vec{p} \approx \hat{\eta}\cdot\vec{p} = \omega^2\,\hat{\eta}\cdot\vec{r}$$

$$\mu_{\mathrm{el}}(\omega) \propto \left|\left\langle \Psi_i^{N-1}\psi_i\right|\,\hat{\eta}\cdot\vec{r}\,\left|\psi_f\,\Psi_f^{N-1}\right\rangle\right|^2$$

Dipole selection rules:

$\Delta\ell = \pm 1$ $\quad\Delta s = 0$

$\Delta j = \pm 1, 0$ $\quad\Delta m = 0$

De-excitation mechanisms

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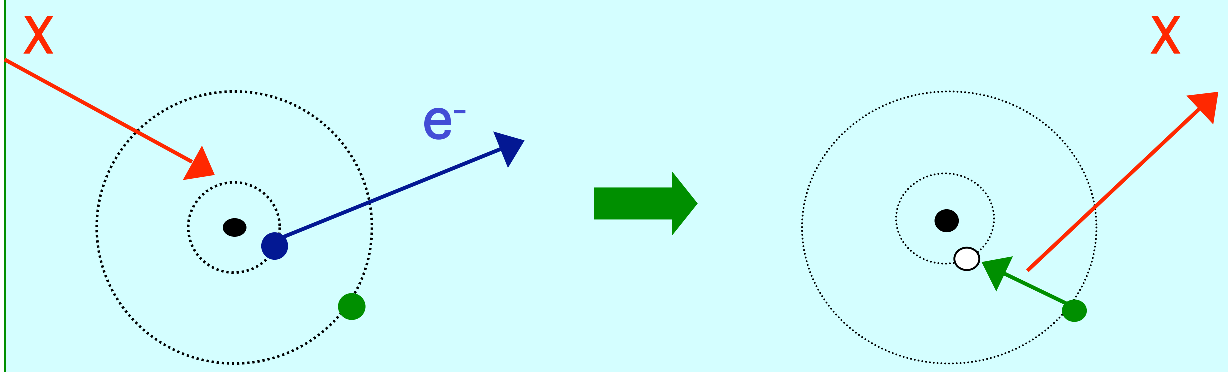
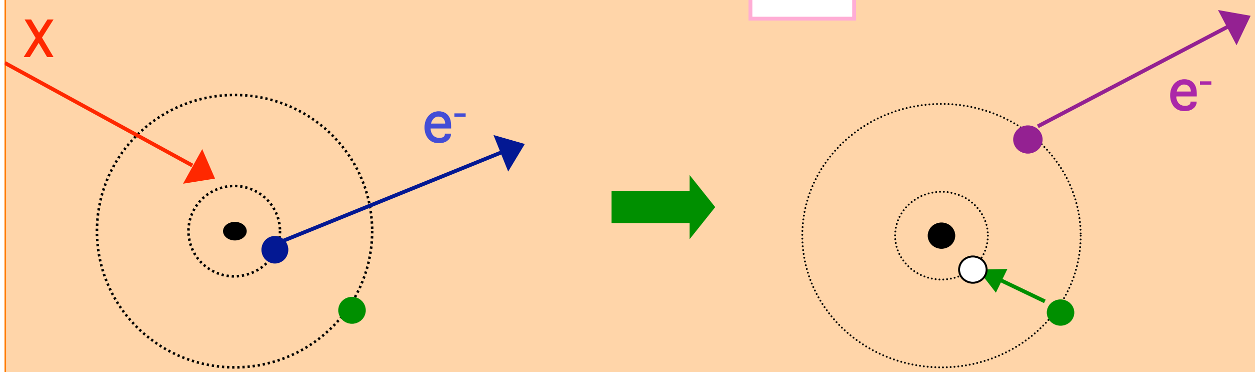
Radiative: fluorescence

X



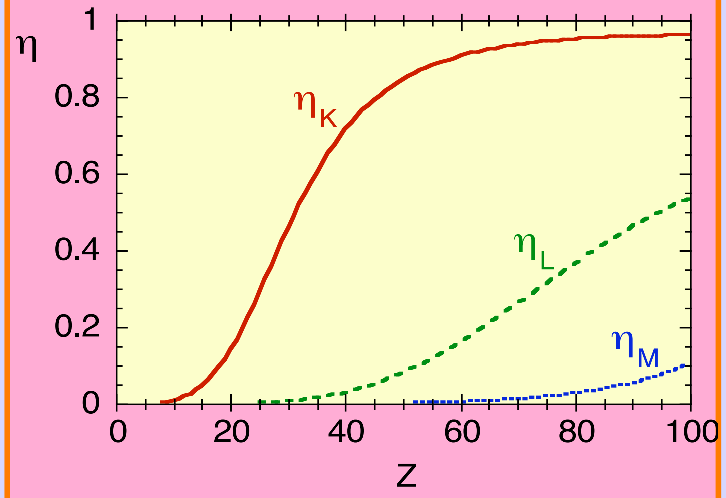
Non-radiative: Auger

A



Fluorescence yield

$$\eta = \frac{X}{X + A}$$



Core-hole lifetime

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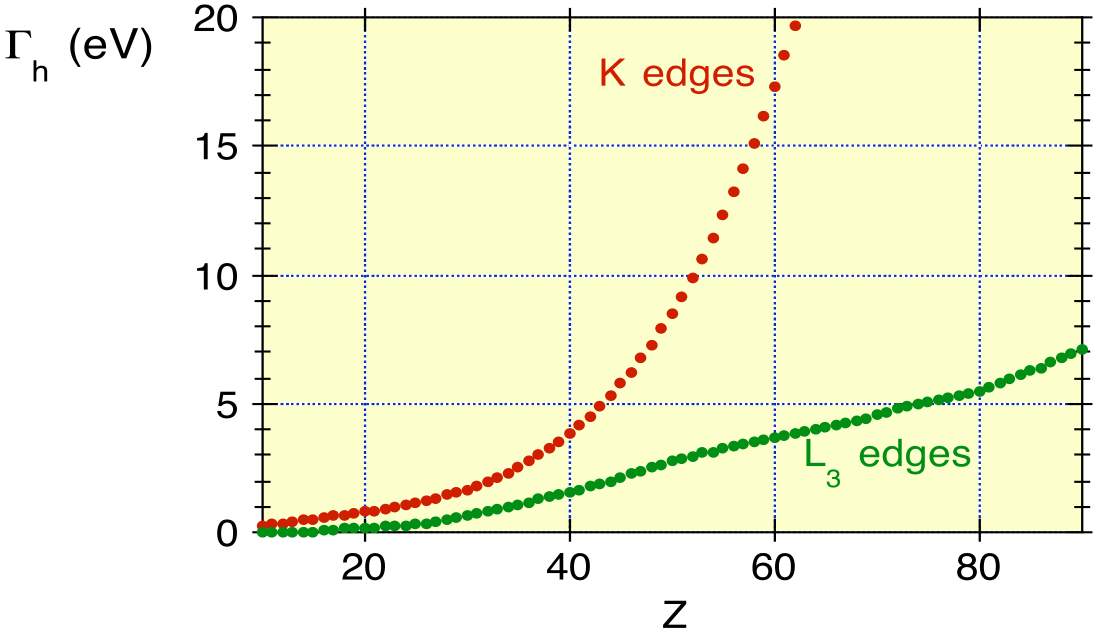
Lifetime of the excited state
$\tau_h \sim 10^{-16} - 10^{-15}$ s

$$\tau_h \approx 1/\Gamma_h$$

Energy width of the excited state
Γ_h



τ_h

Contribution to photo-electron life-time

Γ_h

Energy resolution of XAFS spectra

Summary

- ❑ Attenuation: scattering and absorption
- ➢ Thomson scattering from a free electron (classical)
- ➢ Limits of the classical treatment
- ➢ Basic interference phenomenon
- ➢ Electronic and atomic structure factors
- ➢ Comtpon effect
- ➢ Anomalous (or ‘resonant’) scattering
- ❖ Absorption coefficient and absorption edges
- ❖ Fine structure at absorption edges
- ❖ Introduction to theory of photoelectric absorption


Dolomite mountains at sunset (near Trento, Italy)