Posters - Theory

P30

TILING VERTICES AND THE SPACING DISTRIBUTION OF ITS RADIAL PROJECTIONS

Tobias Jakobi

Department of Mathematics, University of Bielefeld (Germany)
tjakobi@math.uni-bielefeld.de

In [1], Boca, Cobeli and Zaharescu gave an elegant representation of the first consecutive spacing distribution when looking at the visible points of the square lattice $\mathbf{Z}^2$. Here one considers the lattice points which are *visible* from the origin. This amounts to selecting those points **(x,y)** with coprime coordinates. Now place a circle of radius **R** at the origin and project all interior points onto this circle, effectively reducing the polar coordinate description of a point to its angle information. Then sort all these angles and measure the difference between neighbouring ones. In [1], it was proved, even in a more general context, that after proper rescaling there exists a limit distribution of the differences as **R** .

One might ask the question whether the limit distribution encodes relevant information about the degree of order of the input point set. Phrased differently, can one quantify how much the distribution varies when exchanging the original lattice with some other locally finite point set? It is known that the set of Poisson distributed points in the plane yields the exponential distribution, which represents the most *unordered* set. This is one of the few other examples that is fully understood analytically.

Here, we take a look at numerical results for the vertex set of aperiodic tilings in the plane as input (e.g. Ammann-Beenker, rhombic Penrose and chiral Lançon-Billard). In connection with this problem, we also study the visibility property for special aperiodic point sets, generated from a cyclotomic model set description [2]. These cases resemble the lattice situation to some extent (existence of a gap, etc.).

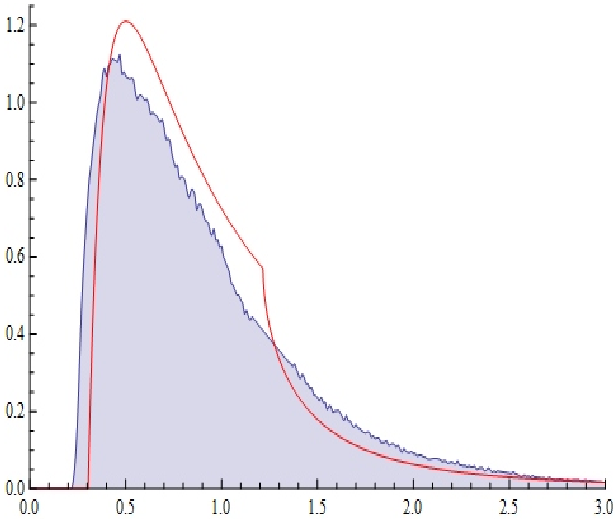

Figure 1. Spacing distribution of a large Ammann-Beenker patch (cyclotomic case).

To evaluate how *robust* the properties of the distributions are, we apply a randomization procedure to the sets which discards a vertex with a fixed probability **p**. It turns out that many properties seem to be continuous in **p**. In particular, the position of the gap is linear, indicating a limit law in the background.

P31

SQUAREFREE NUMBERS AND THEIR DIFFRACTION

Michael Baake, Christian Huck and Tobias Jakobi

Department of Mathematics, University of Bielefeld (Germany)
{mbaake,huck,tjakobi}@math.uni-bielefeld.de

An integer is called *squarefree* if it is not divisible by a nontrivial square. The set of squarefree integers is a discrete subset of the line with gaps of arbitrary size. Nevertheless, it has positive density and a pure point diffraction spectrum [1, 2], as well as other interesting properties as a dynamical system [3, 4].

Recently, the setting was generalized [5] to squarefree numbers in algebraic number fields, where many properties prevail. In this contribution, which complements the tutorial talk [6], some explicit examples are shown in detail. Our emphasis is on the connection with the underlying Minkowski embedding [5].

In particular, we present the diffraction for the squarefree numbers in various rings of integers of quadratic number fields, including the Gaussian integers **Z[i]** as well as **Z[2]** and **Z[ ]**, where is the golden ratio.

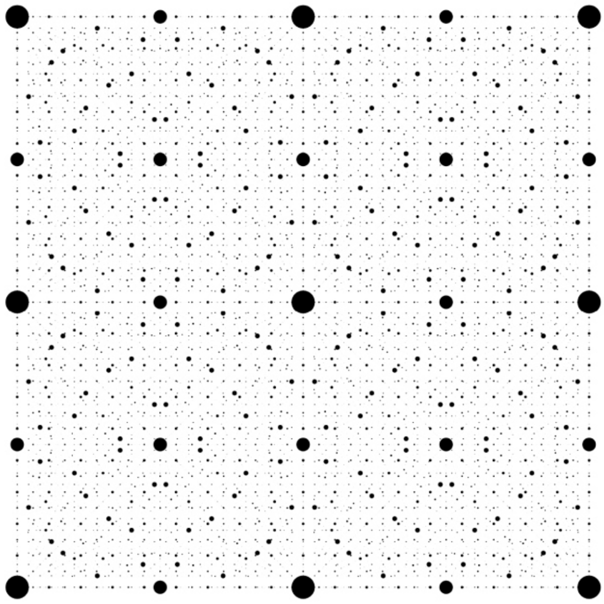

Figure 1. Diffraction of the squarefree Gaussian numbers (non-linear scaling of intensities).

1. M. Baake, R. V. Moody, P. A. B. Pleasants, Diffraction from visible lattice points and k-th power free integers, *Discr. Math.*, **221** (2000), pp. 3-42.
2. M. Baake, U. Grimm, *Aperiodic Order. Vol. I: A Mathematical Invitation*. Cambridge University Press, Cambridge. 2013.
3. F. Cellarosi, Y. G. Sinai, Ergodic properties of square-free numbers, *J. Eur. Math. Soc.*, **15** (2013), pp. 1343-1374.
4. C. Huck, M. Baake, Dynamical properties of k-free lattice points, *Acta Phys. Pol.*, **A126** (2014), pp. 482-485.
5. F. Cellarosi, I. Vinogradov, Ergodic properties of k- free integers in number fields. *J. Mod. Dynam.*, **3** (2013), pp. 461-488.
6. C. Huck, Visible lattice points and weak model sets, *Aperiodic 2015 tutorial talk*.

This work was supported by the German Research Foundation (DFG) within the CRC 701.

P32

DIFFRACTION INTENSITIES OF THE RANDOM NOBLE MEANS SUBSTITUTIONS

Timo Spindeler

Department of Mathematics, University of Bielefeld,33501 Bielefeld, Germany
tspindel@math.uni-bielefeld.de

Although there are many open problems (e.g. the famous Pisot substitution conjecture), the structure of systems with pure point diffraction is rather well understood [1,2]. Additionally, the situation for various systems with diffraction spectra of mixed type has improved [3, 4, 5]. Nevertheless, the understanding of spectra in the presence of entropy is only at its beginning and it is desirable to work out concrete examples.

In 1989, Godrěche and Luck [6] extended the study of conventional substitution rules and introduced the notion of local mixtures of substitution rules on the basis of a fixed probability vector along the random Fibonacci substitution. They presented first results concerning the topological entropy and the spectral type of the diffraction measure of associated point sets. This was further developed in [7]. Here, we are interested in the pure point part of the diffraction pattern.

The aim is to present a generalisation of this concept by regarding the so-called *noble means families*, see also [7], each consisting of finitely many primitive substitution rules that individually all define the same two-sided discrete dynamical hull, and to determine a closed expression for the diffraction intensities of their randomised versions. To do this, we will consider an equation system of self-similar measures of Hutchinson type, defined by a compact family of contractions. The solution of this equation system gives the formula of the diffraction intensities.

1. M. Baake, R.V. Moody, Weighted Dirac combs with pure point diffraction, *J. reine angew. Math. (Crelle)* **573** (2004), pp. 61-94.
2. M. Queffélec, *Substitution Dynamical Systems. Spectral Analysis*, 2nd ed. Springer, Berlin (2010).
3. M. Baake, M. Birkner, R.V Moody, Diffraction of stochastic point sets: Explicitly computable examples, *Commun. Math. Phys.* **293** (2010), 611-660.
4. M. Baake, F. Gähler, U. Grimm, Spectral and topologicalproperties of a family of generalised Thue-Morse sequences, *J. Math. Phys.* **53** (2012), 032701 (24pp).
5. M. Baake, U. Grimm, *Aperiodic Order. Volume 1: A Mathematical Invitation*, Cambridge University Press, Cambridge (2013).
6. C. Godreche, J. M. Luck, Quasiperiodicity and randomness in tilings of the plane, *J. Stat. Phys*. **55** (1989), pp.1-28.
7. M. Moll, Diffraction of random noble means words, *J. Stat. Phys*. **156** (2014), pp.1221-1236.

This work is supported by the German Research Foundation (DFG) via the Collaborative Research Centre (CRC 701) through the faculty of mathematics, University of Bielefeld.

P33

DIFFRACTION OF A SIMPLE NON-PISOT INFLATION

M. Baake[1], U. Grimm[2]

[1]Fakultät für Mathematik, Universität Bielefeld, Postfach 100131, 33501 Bielefeld, Germany
[2]Department of Mathematics and Statistics, The Open University, Milton Keynes MK7 6AA, UK
uwe.grimm@open.ac.uk

Point sets derived from substitution systems form an important class of aperiodically ordered structures. There are many examples for pure point diffractive substitution systems, such as the ubiquitous Fibonacci chain and its generalisations, but we also have paradigms for structures with singular continuous diffraction (the Thue–Morse chain) and absolutely continuous diffraction (the Rudin–Shapiro chain). One can quite easily combine these and produce examples with mixed diffraction spectrum including any combinations of these three components [1].

According to the Pisot substitution conjecture, primitive substitutions with a substitution matrix whose characteristic polynomial is irreducible and whose eigenvalues, apart from the leading eigenvalues, are less than one in modulus (in which case the inflation multiplier is a Pisot–Vijayaraghavan or short PV number) show pure point spectrum. While there is no proof of this conjecture, there are no counterexamples known [2], and it is widely believed to hold.

There is also a good understanding of substitutions of constant length, both in one and in higher dimensions [3–5]. This is due to the fact that, in the constant length case, the symbolic side and the geometric realisation with tiles of natural size coincide, because all tiles have equal length or are congruent. This also leads to a rather direct relation between the diffraction measures of the system (and systems derived as images of sliding block maps) on the one hand and the dynamical spectral measures on the other [6].

So far, much less is known for non-Pisot substitutions. Here, we concentrate on a particular example [7], the primitive two-letter substitution *aa* *aaaaaaa* and *bb* *aa* with inflation multiplier = (1 + 13)/2. For the one-dimensional point set corresponding to the appropriate geometric realisation in terms of two intervals of length and 1, we sketch an argument that indicates that the diffraction, apart from a trivial Bragg peak at the origin, is singularly continuous.

1. M. Baake, U. Grimm, *Aperiodic Order. Volume 1: A Mathematical Invitation*. Encyclopedia of Mathematics and its Applications No. 149. Cambridge: Cambridge University Press. 2013.
2. S. Akiyama, F. Gähler, J.-Y. Lee, Determining pure discrete spectrum for some self-affine tilings, *Disc. Math. Th. Comp. Sci.* (DMTCS), **16:3**, (2014), 305–316; arXiv:1403.0362.
3. M. Queffélec, Substitution Dynamical Systems — Spectral Analysis. LNM 1294, 2nd ed. Berlin: Springer. 2010.
4. N.P. Frank, Multi-dimensional constant-length substitution sequences, *Topol. Appl.,* **152**, (2005), 44–69.
5. A. Bartlett, Spectral theory of substitutions in , *preprint* arXiv:1410.8106.
6. M. Baake, D. Lenz, A. C. D. van Enter, Dynamical versus diffraction spectrum for structures with finite local complexity, Ergodic Th. & Dynam. Syst., in press; arXiv:1307.7518.
7. M. Baake, N. P. Frank, U. Grimm, E. A. Robinson, Diffraction of a non-Pisot inflation point set and absence of absolutely continuous spectrum, in preparation.

P34

SHAPE LIMIT IN VORONOI TILINGS FOR BERNOULLI SPIRALS

Y. Yamagishi[1], T. Sushida[2], A. Hizume[1]

[1]Department of Applied Mathematics and Informatics, Ryukoku University, Seta, Otsu 520-2194, Japan
[2]Organization for the Strategic Coordination of Research and Intellectual Property, Meiji University, 4-21-1 Nakano, Tokyo 164-8525, Japan
yg@rins.ryukoku.ac.jp

Let $z^k : k \quad Z \quad C$ be a Bernoulli spiral set in the complex plane, generated by $z \quad r\exp(i \;_k)$, $0 \quad r \quad 1$. We studied the geometry and topology of triangular tilings with the vertex set in [1], and the shape limit of triangular tiles as r 1 in [2]. In the phyllotaxis theory, $1/r$ is called the *plastochrone ratio*, and the *divergence angle*. Here we consider the Voronoi tiling with the site set . The *parastichy* number, i.e. the number of spirals consisting of contact Voronoi cells, is obtained by the continued fraction expansion of the . The Voronoi tiling is a quadrilateral tiling if it has two parastichies, or hexagonal tiling if it has three parastichies.

Suppose that /2 is a quadratic irrational number. If we only consider the quadratic Voronoi tilings, then the limit set of the shapes of the quadrilateral tiles as r 1 is a finite set of rectangles. In particular, if /2 is linearly equivalent to the golden section = (1 + 5)/2, the limit is the square [3].

Rothen and Koch [4] observed the *shape invariance under compression* with the golden section divergence angle, in the linear lattice model. Our work is an extension to the cylindrical model. The shape limit in the linear lattice model was studied in [5].

1. T. Sushida, A. Hizume, Y. Yamagishi, *J. Phys. A: Math. Theor.*, **45**, (2012), 235203.
2. T. Sushida, A. Hizume, Y. Yamagishi, *Acta Phys. Polonica A*, **126**, (2014), 633-636.
3. Y. Yamagishi, T. Sushida, A. Hizume, *Nonlinearity*, **28**, (2015),1077-1102.
4. F. Rothen, A.-J. Koch, *J. Physique France*, **50**, (1989), 633-657.
5. T. Sushida, A. Hizume, Y. Yamagishi, *RIMS Kokyuroku Bessatsu*, **B47**, (2014), 023-032.

This work is partially supported by JSPS Kakenhi Grant 24654029, 15K05011, and the Joint Research Center for Science and Technology of Ryukoku University. We would like to thank Smith College for the hospitality extended to Yamagishi during his stay in 2015-2016.

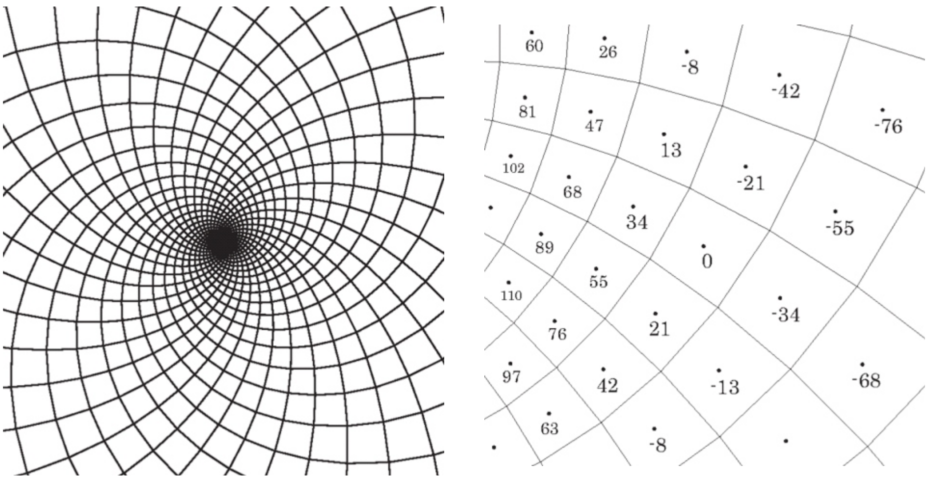

Figure 1. A quadrilateral Voronoi tiling with similarity symmetry, generated by z = (0,999426) exp (2 i = (1 + 5)/2. A global view and a local view. Each site point is indexed by an integer. The tiles are close to squares.

P35

ENGINEERING SPECTRAL CROSSOVER AND FLAT BAND STATES IN APERIODICALLY DISTORTED LADDER NETWORKS

Arunava Chakrabarti

Department of Physics, University of Kalyani, Kalyani, West Bengal 741235, India
arunava_chakrabarti@yahoo.co.in

We discuss the nature of electronic spectrum of a class of tight binding aperiodic ladder networks using real space renormalization methods. The ladder (Fig. 1) is described by the Hamiltonian:

$$H \quad {}_{n} |n\rangle\langle n| \quad t_{nm} |n\rangle\langle m| \qquad (1)$$
$$n \qquad\qquad n,m$$

Using a simple change of basis we decouple the Schroedinger equation for the ladder into a set of two difference equations as,

$$[E \;\; (\;{}_{n} \quad {}_{n})] \;\; {}_{n,A} \quad (t_{n,n \; 1} \quad {}_{n}) \;\; {}_{n \; 1,A} \qquad (2)$$
$$(t_{n,n \; 1} \quad {}_{n \; 1}) \;\; {}_{n \; 1,A}$$

$$[E \;\; (\;{}_{n} \quad {}_{n})] \;\; {}_{n,B} \quad (t_{n,n \; 1} \quad {}_{n}) \;\; {}_{n \; 1,B}$$
$$(t_{n,n \; 1} \quad {}_{n \; 1}) \;\; {}_{n \; 1,B}$$

We then introduce *aperiodicity* in the on-site potentials ${}_{n}$ and then, sequentially, in hopping integrals t_{nm} along the arms of the ladder and in between them (${}_{n}$) *to simulate a quasi-periodic distortion* in ladder geometry. *Aperiodic modulations* of the form ${}_{n}$ = ${}_{0}$ cos(Q*n a* +) and its variants [1] are considered for the *on-site potentials* as well as for the *intra-arm*, *inter-arm* and *diagonal* hopping integrals, simulating a distortion in ladder geometry. It is seen that, suitably chosen correlations between the parameters, strongly supported by a *non-trivial role of the phase factors* in each case can result in a *distortion-driven metal-insulator crossover* in the spectral properties (Fig. 2) as well as *flat, dispersionless bands* that are of much current interest in view of the *topological states* [2]. Such modulations are possible in the formation of *optical lattices* and the study thus opens up the possibility of encountering exotic electronic states in quasi-one dimensions.

1. S. Ganeshan, K. Sun and S. Das Sarma, *Phys. Rev. Lett.*, **110**, (2013), 180403.
2. C. Danieli, J. D. Bodyfelt, and S. Flach, arXiv:1502:06690, (2015).

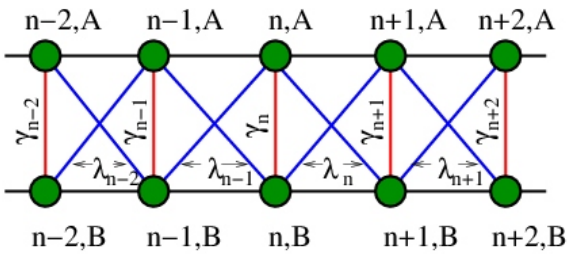

Figure 1. Portion of an infinite aperiodic ladder network.

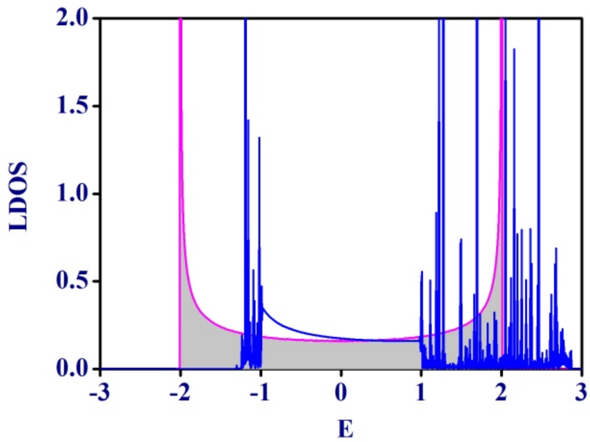

Figure 2. Local density of states of the two decoupled arms showing the possibility of a distortion-driven metal-insulator crossover in the ladder spectrum.

P36

SIMILAR SUBMODULES AND COINCIDENCES OF CUBIC MODULES

P. Zeiner

Faculty of Mathematics, Bielefeld University, Box 100131, 33501 Bielefeld, Germany
pzeiner@math.uni-bielefeld.de

Similar sublattices and coincidence site lattices in low dimensions are quite well understood [1, 2]. Moreover, similar submodules and coincidence site modules (CSMs) of certain planar modules with n-fold symmetry have been studied in detail [3, 4]. For n =5,8,10,12, these modules correspond to certain algebraic polynomials of degree four, whereas planar lattices correspond to quadratic polynomials. Here, we want to discuss certainmodules of rank 3, corresponding to cubic algebraic polynomials. We investigate their similar submodules and CSMs and the connections between them [5]. It turns out that several new phenomena occur. We illustrate them by several examples, including modules corresponding to the polynomial $x^3 \quad 3x \quad 1$.

1. M. Baake, R. Scharlau, P. Zeiner, Similar sublattices of planar lattices, *Canad. J. Math.*, **63**, (2011), 1220-1237, arXiv:0908.2558.
2. M. Baake, Solution of the coincidence problem in dimensions $d \quad 4$, in *The Mathematics of Long-Range Aperiodic Order*, edited by R.V. Moody (Dordrecht: Kluwer),1997, pp. 9-44, arXiv:math.MG/0605222.
3. M. Baake, U. Grimm, Bravais colourings of planar modules with N-fold symmetry, *Z. Kristallogr.,* **219**, (2004), 72-80, arXiv:math.CO/0301021.
4. P.A.B. Pleasants, M. Baake, J. Roth, Planar coincidences with N-fold symmetry, *J. Math. Phys.,* **37**, (1996), 1029-1058, arXiv:math.MG/0511147.5. S. Glied, Similarity and coincidence isometries for modules, *Can. Math. Bull.,* 55, (2011), 98-107, arXiv:1005.5237.
5. S. Glied, Similarity and coincidence isometries for modules, *Can. Math. Bull.,* **55**, (2011), 98-107, arXiv:1005.5237.

P37

COINCIDENCE SITE PATTERNS IN THE PINWHEEL TILING

Reinhard Lück

Weilstetter Weg 16, D-70567 Stuttgart, Germany
r.v.lueck@web.de

Coincidence site lattices (CSLs) have been investigated for periodic as well as for quasiperiodic lattices in two and higher dimensions since some decades. They arise by a rotation of a lattice with respect to a copy of itself and can almost be characterized by the so-called Sigma-value ( ), which indicates the reciprocal value of the density of coinciding lattice points. We investigated empirically coincidences in the pinwheel tiling [1] caused by rotation as well as by shift or reflection. For rotation angles 2 arctan(m/n) with $m^2 + n^2 = k\ 5^p$, $k = 1, 2;\ p = 0, 1, 2, \ldots$ coinciding tiles and patches of coinciding tiles were generated. In contrast to CSLs of other tilings, these coinciding tiles are distributed inhomogeneously and anisotropically. No analogous to the Sigma value does exist; the density of coinciding tiles is discussed for a few examples using the generating substitution process.

1. M. Baake and U. Grimm, *Aperiodic Order,* Volume 1 (Cambridge University Press), 2013, p 224.

This contribution is prepared in collaboration with David H. Warrington and dedicated to him on the occasion of his 80th birthday.

P38

AN ICOSAHEDRAL QUASICRYSTAL AS A GOLDEN MODIFICATION OF THE ICOSAGRID AND ITS CONNECTION TO THE E8 LATTICE

Fang Fang, and Klee Irwin

Quantum Gravity Research, Los Angeles, CA, U.S.A.
fang@quantumgravityresearch.org

We present an icosahedral quasicrystal as a modification of the icosagrid, a multigrid with 10 plane sets that arranged with icosahedral symmetry. We use the Fibonacci chain to space the planes obtaining a quasicrystal with icosahedral symmetry. It has a surprising correlation to the Elser-Sloane quasicrystal [4], a 4D cut-and-project of the E8 lattice. We call this quasicrystal the Fibonacci modi?ed icosigrid quasicrystal(FMIQ). We found that the FMQC totally imbeds another quasicrystal that is a compound of 20 3D slices of the Elser-Sloane quasicrystal. The slices, which contains only regular tetrahedra, are put together by a certain 'golden rotation' [5]. Interesting 20Gs (20-tetrahedron clusters arranged with the 'golden rotation') appear repetitively in the FMQC and are arranged with icosahedral symmetry. It turns out that this 'golden rotation' is the dihedral angle of the 600-cell (the super-cell of the Elser-Sloane quasicrystal) and the angel between the tetrahedral facets in the E8 polytope known as the Gosset polytope. We suggest that the FMIQ is an alternative result of releasing the transdimensional "geometric frustration" while maintaining the regularity of the tetrahedra [6-10].

1. I. Brovchenko and A. Oleinikova. Multiple phases of liquid water. *Chem Phys Chem*, **9**(18):2660– 2675, 2008.
2. P. F. Damasceno, M. Engel, and S. C. Glotzer. Predictive self-assembly of polyhedra into complex structures. *Science*, **337**(6093):453–457, 2012.
3. V. Dmitrienko and M. Kl´eman. Tetrahedral structures with icosahedral order and their relation to quasicrystals. *Crystallography Reports*, **46**(4):527–533, 2001.
4. V. Elser and N. J. A. Sloane. A highly symmetric four-dimensional quasicrystal. *J. Phys. A*, **20**(18):6161–6168, 1987.
5. F. Fang, K. Irwin, J. Kovacs, and G. Sadler. Cabinet of curiosities: the interesting geometry of the angle = arccos((3 - 1)/4). ArXiv e-prints, 1304.1771, Apr. 2013.
6. A. Haji-Akbari, M. Engel, A. S. Keys, X. Zheng, R. G. Petschek, P. Palfy-Muhoray, and S. C. Glotzer. Disordered, quasicrystalline and crystalline phases of densely packed tetrahedra. *Nature*, **462**:773–777, 2009.
7. R. V. Moody and J. Patera. Quasicrystals and icosians. *J. Phys. A*, **26**:2829–2853, 1993.
8. J. Peters and H.-R. Trebin. Tetracoordinated quasicrystals. *Phys. Rev. B*, **43**:1820–1823, Jan 1991.
9. C. L. Phillips and S. C. Glotzer. Effect of nanoparticle polydispersity on the self-assembly of polymer tethered nanospheres. *The Journal of Chemical Physics*, **137**(10):104901, 2012.
10. C. L. Phillips, E. Jankowski, M. Marval, and S. C. Glotzer. Self-assembled clusters of spheres related to spherical codes. *Phys. Rev. E*, **86**:041124, Oct 2012.

P39

OCTAGONAL TYPE OF THE QUASIPERIODIC SUCCESSION ALGORITHM

U. Gaenshirt[1], M. Willsch[2]

[1]*Sculptor and Researcher, Wartburgstr. 2, D-90491 Nuremberg, Germany*
[2]*Physicist, Siemens AG, D-91050 Erlangen, Germany*
uli.gaenshirt@yahoo.de

The *decagonal quasiperiodic succession algorithm* [1], related to *decagonal cluster cells* [2], generates the growth of an infinite *cartwheel-type tiling*, although it acts locally.

The paper presents a new version type, applicable for coverings of *octagonal clusters cells* $\boldsymbol{Q}$ (Fig. 1a) which have an equivalent relation to the *Gähler octagons* in a perfect *Ammann-Beenker tiling* [3]. The cell $\boldsymbol{Q}$ is based on the quasiperiodic *Ammann 8-grid* , a superposition of four 1D-grids $^{\mathbf{a}}$, $^{\mathbf{b}}$, $^{\mathbf{c}}$, $^{\mathbf{d}}$. The used substitution factor of is ^{2} (silver mean = 1+ 2). The growth process is controlled by the scale values **a**, **b**, **c**, **d** of the twin-scales $\mathbf{I}^{\mathbf{a}\pm}$, $\mathbf{I}^{\mathbf{b}\pm}$, $\mathbf{I}^{\mathbf{c}\pm}$, $\mathbf{I}^{\mathbf{d}\pm}$ (in general $\mathbf{I}^{x\pm}$) which are fixed on the cell grid ^{Q} in a specific relation. On both scales $\mathbf{I}^{x+}$ and $\mathbf{I}^{x-}$ of a twin-scale $\mathbf{I}^{x\pm}$ two identical values $\boldsymbol{x}$, with $\boldsymbol{x}$ $\{\boldsymbol{x}^{def}\}$, are synchronised by a sliding ruler. Its length, $\mathbf{L}^{aver}$, is the average of the ***q**-line grid intervals* $\mathbf{L}^{q}$ and $\mathbf{S}^{q}$, with respect to the ratio 2:1 of their lengths and 1: 2 of their frequency rate in an infinitely expanded grid ^{q,} .

The *octagonal quasiperiodic succession algorithm* distinguishes 7 neighbour transformations $\boldsymbol{h}_{\boldsymbol{k}}(\boldsymbol{Q})$ with 4 specified equations each. The algorithm correlates the twin-scales of a cell $\boldsymbol{Q}$ with the parallel twin-scales of a cell

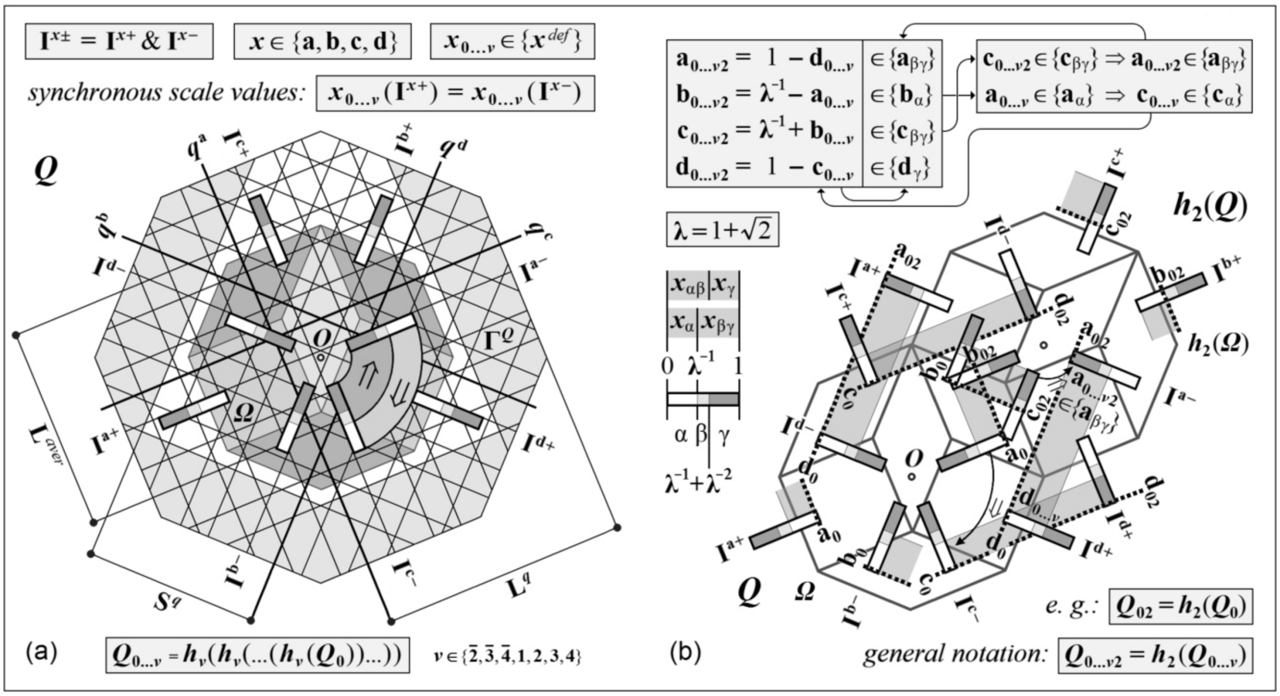

Figure 1. (a) Cluster cell $\boldsymbol{Q}$ with four twin-scales $\mathbf{I}^{x\pm}$, (b) Twin-scale correlation of cluster cells $\boldsymbol{Q}$ and $\boldsymbol{h}_2(\boldsymbol{Q})$.

$\boldsymbol{h}_k(\boldsymbol{Q})$, converts their values and then verifies or falsifies the transformation. A *verified* transformation (e.g. Fig. 1b) will be denoted $\boldsymbol{h}_v(\boldsymbol{Q})$. Beginning with a *start-cell* $\boldsymbol{Q}_0$ only cells of the form $\boldsymbol{Q}_{0\ldots v} = \boldsymbol{h}_v(\boldsymbol{h}_v(\ldots(\boldsymbol{h}_v(\boldsymbol{Q}_0))\ldots))$ are realized.

As a result we propose a recursive 7x4-*formula set* generating a flawless infinite step-by-step growth of an *octagonal Ammann-Beenker substitution tiling*, solely using local information.

1. U. Gaenshirt, M. Willsch, *Philos. Mag.*, **87**, (2007), 3055-3065.
2. P. Gummelt, *Geometriae Dedicata*, **62**, (1996), 1-17.
3. S. I. Ben Abraham, F. Gähler, *Phys. Rev.*, B **60**, (1999), 860-864.

P40

STRUCTURE FACTOR FOR GENERALIZED PENROSE TILING

M. Chodyń[1], P. Kuczera[1,2] and J. Wolny[1]

[1]Faculty of Physics and Applied Computer Science, AGH University of Science and Technology, Mickiewicz Avenue 30, 30-059 Krakow, Poland
[2]Laboratory of Crystallography, ETH Zurich, Wolfgang-Pauli-Strasse 10, Zurich, CH-8093, Switzerland
maciej.chodyn@gmail.com

The Generalized Penrose Tiling (GPT) [1,2] can be considered a promising alternative for Penrose Tiling (PT) as a model for decagonal quasicrystal refinement procedure, particularly in the statistical approach (also called the Average Unit Cell (AUC) approach) [3]. The statistical method using PT has been successfully applied to the structure optimization of various decagonal phases [4]. The application of the AUC concept to the GPT will be presented.

In the higher dimensional (*n*D) approach, PT is obtained by projecting a 5D hypercubic lattice through a window consisting of four pentagons, called the atomic surfaces (ASs), in the perpendicular space. The vertices of these pentagons together with two additional points form a rhombicosahedron. The GPT is created by projecting the 5D hypercubic lattice through a window consisting of five polygons, generated by shifting the ASs along the body diagonal of the rhombicosahedron. Three of the previously pentagonal ASs will become decagon, one will remain pentagonal and one more pentagon will be created (for PT it is a single point). The structure of GPT will depend on the shift parameter, its building units are still thick and thin rhombuses, but the matching rules and the tiling changes. Diffraction pattern of GPT have peaks in the same positions as regular PT, however their intensities are different.

Binary decagonal quasicrystal structure with arbitrary decoration for a given shift value was simulated. Its diffraction pattern was calculated using AUC method [5,6]. Generated diffraction pattern were used as “experimental data set” in structure refinement algorithm made to test the refining of shift parameter.

1. K. N. Ishihara, A. Yamamoto, *Acta Cryst.* (1988), A**44**, 508-516.
2. Pavlovitch, M. Kleman, *J Phys a-Math Gen.* (1987), **20**, 687-702.
3. J. Wolny, *Phil. Mag.* (1998), **77**, 395-414.
4. P. Kuczera, J. Wolny, W. Steurer, *Acta Cryst.* (2012), **B68**, 578–589.
5. B. Kozakowski, J. Wolny, *Acta Cryst* (2010), **A66**, 489-498.
6. M. Chodyn, P. Kuczera, J. Wolny, *Acta Cryst*. (2015), **A71**, 161–168.

P41

ELECTRONIC TRANSPORT IN TEN MOST STUDIED APERIODIC SYSTEMS

F. Sánchez[1], C. Ramírez[1], V. Sánchez[1], C. Wang[2]

[1] *Departamento de Física, Facultad de Ciencias, Universidad Nacional Autónoma de México, Apartado Postal 70-542, 04510 México D.F., México*
[2] *Instituto de Investigaciones en Materiales, Universidad Nacional Autónoma de México, Apartado Postal 70-360, 04510 México D.F., México*
odiseo.f27@gmail.com

The electronic transport in solids with a large number of impurities is still an unclear issue, where the interference between the electronic wavefunction and aperiodic potentials has multiple consequences. Due to the presence of such impurities, the translational symmetry is lost making the reciprocal lattice method inadequate or useless. In consequence, the most studies of aperiodic systems have been carried out in finite clusters with or without the periodic boundary condition. The former frequently introduces undesirable contributions derived from the artificial periodic boundary condition and the latter over emphasizes the molecular character of discrete energy spectra. In this work, we use a full real-space renormalization plus convolution method [1] to study the electronic transport in aperiodic macroscopic systems with bond disorder under constant or oscillating electric fields. This method has the advantage of being computationally efficient, making able to address macroscopic aperiodic systems without introducing extra approximations [2]. We present numerical and analytical results of dc and ac electrical conductivities for ten most studied aperiodic systems, such as generalized Fibonacci, Thue-Morse, period-doubling, triadic Cantor, Rudin-Shapiro and paper folding sequences [3]. In particular, analytical studies reveal more than one transparent state in the Thue-Morse, period-doubling and triadic Cantor systems. Moreover, the dc conductivity spectra show narrow zones with almost ballistic transport when sequences are not quasiperiodic. Finally, a comparative analysis of the ac conductivity in these systems is also presented.

This work has been partially supported by UNAM-IN113813, UNAM-IN113714 and CONACyT- 131596. Computations were performed at Miztli of DGTIC-UNAM.

1. V. Sánchez, C. Wang, *Physical Review B*, **70**, (2004), 144207.
2. C. Wang, F. Salazar and V. Sanchez, *Nano Lett.*, **8**, (2008) 4205.
3. E. Macia, *Rep. Prog. Phys*., **69**, (2006), 397–441.

P42

APERIODIC TILINGS FROM HIGHER DIMENSIONAL LATTICES

Mehmed Koca

Sultan Qaboos University, College of Science, Department of Physics, Al-Khod 123, PO Box36, Muscat, Oman

A general group theoretical discussion on the projection of the higher dimensional lattices described by the affine Coxeter-Weyl groups is presented. When the lattices are projected onto the Coxeter plane it is noted that the maximal dihedral subgroup *Dh* with *h* representing the Coxeter number describes the *h*-fold symmetric aperiodic tilings. A number of examples have been presented for the groups *B*4, *F*4, *B*5, *B*6 and *E*6 describing the 8 fold, 10 fold, 12 fold symmetries. Projection of the hypercubic lattice B6 into 3D with icosahedral symmetry is discussed.